Data Structure and Algorithm

Heap

Heap

- A heap is a complete binary tree except the bottom level adjusted to the left.
- The value of each node is greater than that of its two children. (Max Heap)
- The value of each node is less than that of its two children. (Min Heap)
- Height of the heap is $\log_2 n$.
- Example

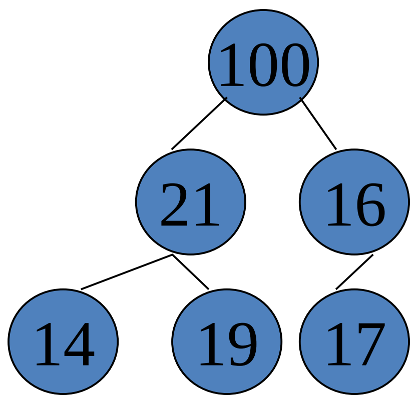

Figure: Not a Heap

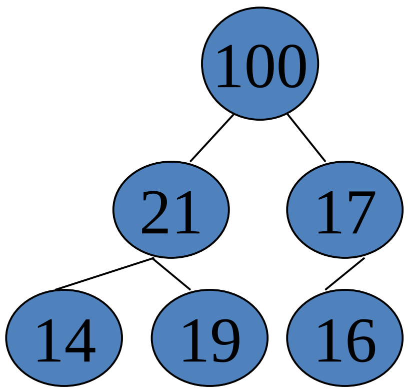

Figure: A Heap

Heap Implementation

- We can use an array (due to the regular structure or completeness of binary tree).
- For a node N with location i, the following factors can be calculated.

  1. Left child of N is in location (2 * i).

  2. Right child of N is in location (2 * i + 1).

  3. Parent of N is in location [i/2].

- Example

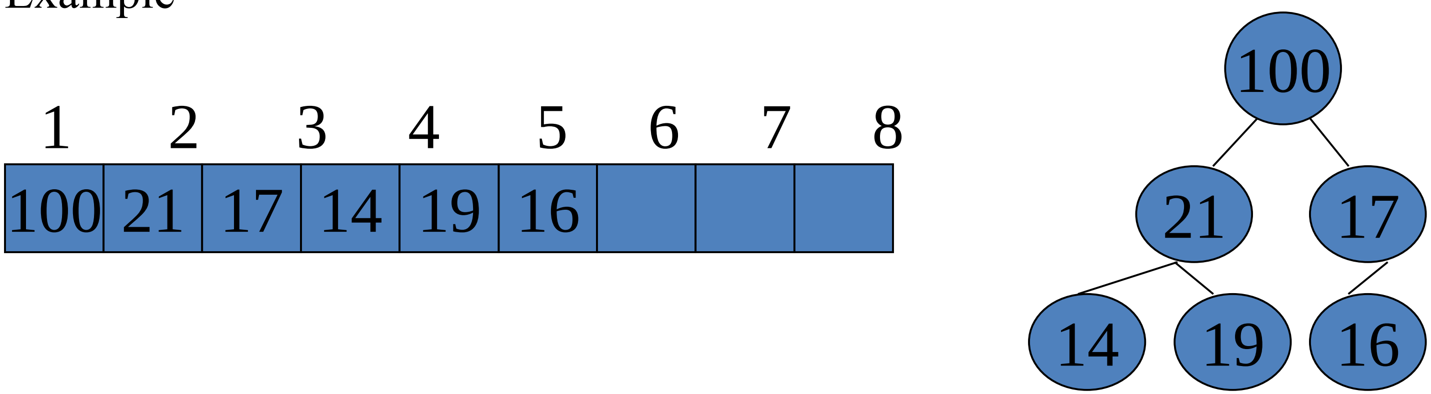

Figure: Heap and Its Array Representation

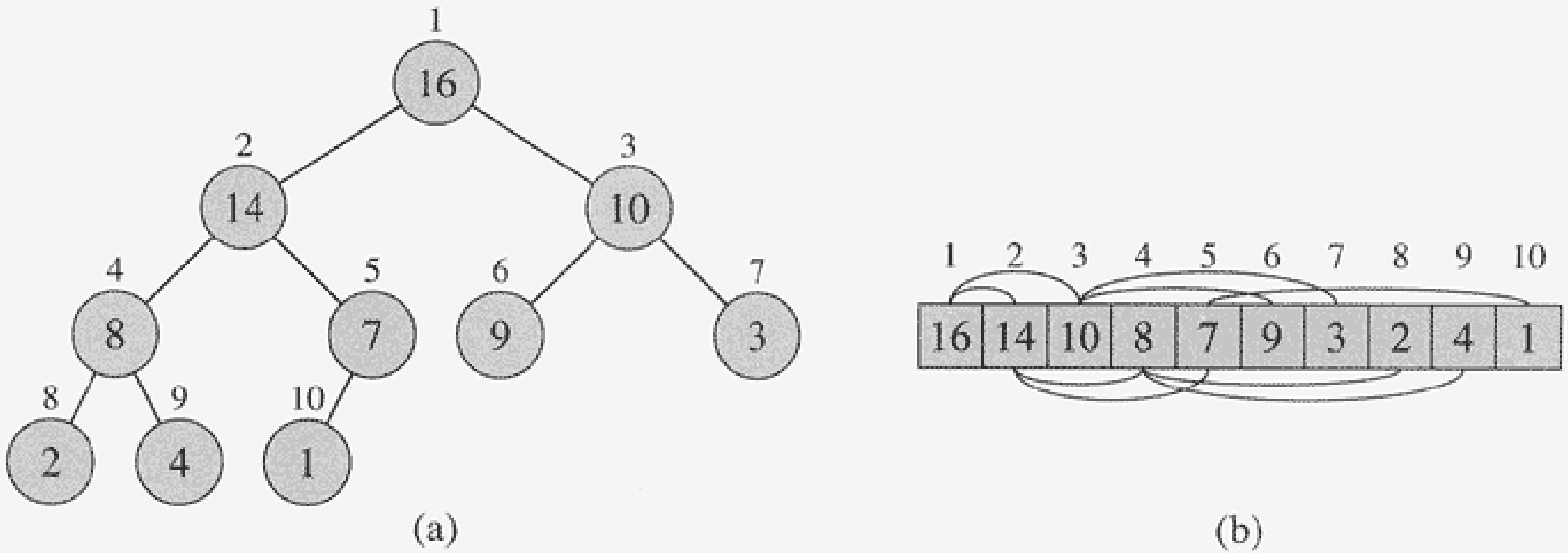

Figure 6.1 A max-heap viewed as **(a)** a binary tree and **(b)** an array. The number within the circle at each node in the tree is the value stored at that node. The number above a node is the corresponding index in the array. Above and below the array are lines showing parent-child relationships; parents are always to the left of their children. The tree has height three; the node at index 4 (with value 8) has height one.

```
MAX-HEAPIFY(A, i)
l ← LEFT(i)
r ← RIGHT(i)
if l ≤ heap-size[A] and A[l] > A[i]
    then largest ← l
    else largest ← i
if r ≤ heap-size[A] and A[r] > A[largest]
    then largest ← r
if largest ≠ i
    then exchange A[i] ↔ A[largest]
         MAX-HEAPIFY(A, largest)
```

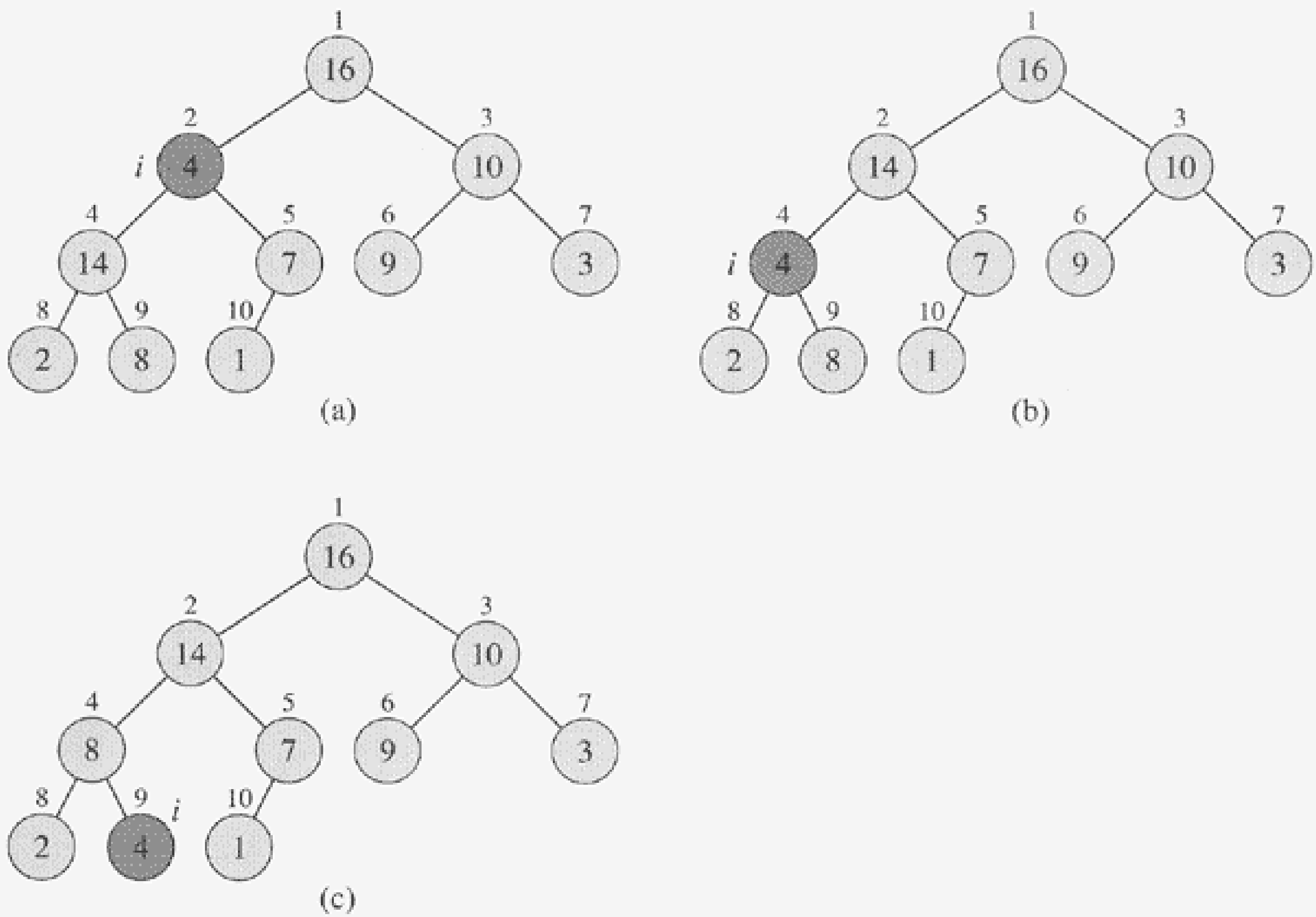

Figure 6.2 The action of MAX-HEAPIFY(A, 2), where *heap-size*[A] = 10. **(a)** The initial configuration, with $A[2]$ at node $i = 2$ violating the max-heap property since it is not larger than both children. The max-heap property is restored for node 2 in **(b)** by exchanging $A[2]$ with $A[4]$, which destroys the max-heap property for node 4. The recursive call MAX-HEAPIFY(A, 4) now has $i = 4$. After swapping $A[4]$ with $A[9]$, as shown in **(c)**, node 4 is fixed up, and the recursive call MAX-HEAPIFY(A, 9) yields no further change to the data structure.

```
BUILD-MAX-HEAP(A)
heap-size[A] ← length[A]
for i ← ⌊length[A]/2⌋ downto 1
        do MAX-HEAPIFY(A, i)
```

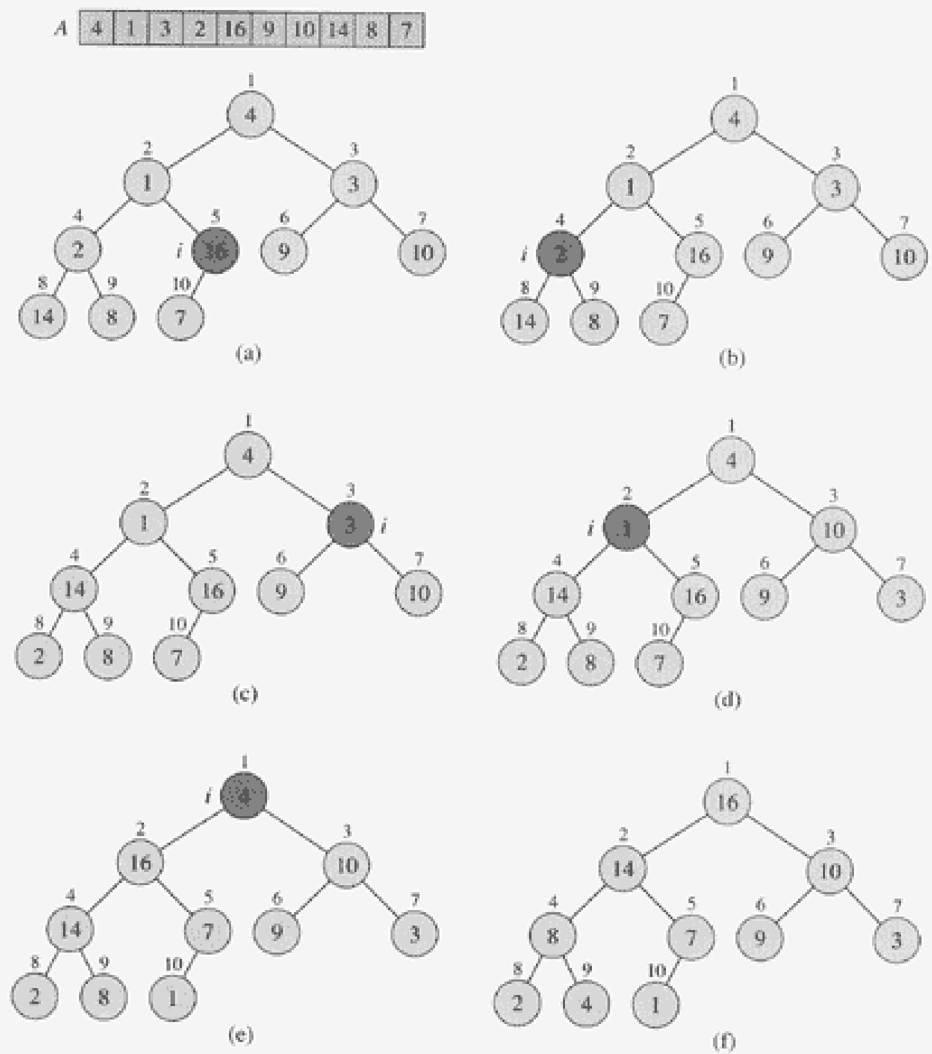

Figure 6.3 The operation of BUILD-MAX-HEAP, showing the data structure before the call to MAX-HEAPIFY in line 3 of BUILD-MAX-HEAP. **(a)** A 10-element input array A and the binary tree it represents. The figure shows that the loop index i refers to node 5 before the call MAX-HEAPIFY(A, i). **(b)** The data structure that results. The loop index i for the next iteration refers to node 4. **(c)–(e)** Subsequent iterations of the **for** loop in BUILD-MAX-HEAP. Observe that whenever MAX-HEAPIFY is called on a node, the two subtrees of that node are both max-heaps. **(f)** The max-heap after BUILD-MAX-HEAP finishes.

```
HEAPSORT(A)
  BUILD-MAX-HEAP(A)
  for i ← length[A] downto 2
      do exchange A[1] ↔ A[i]
         heap-size[A] ← heap-size[A] − 1
         MAX-HEAPIFY(A, 1)
```

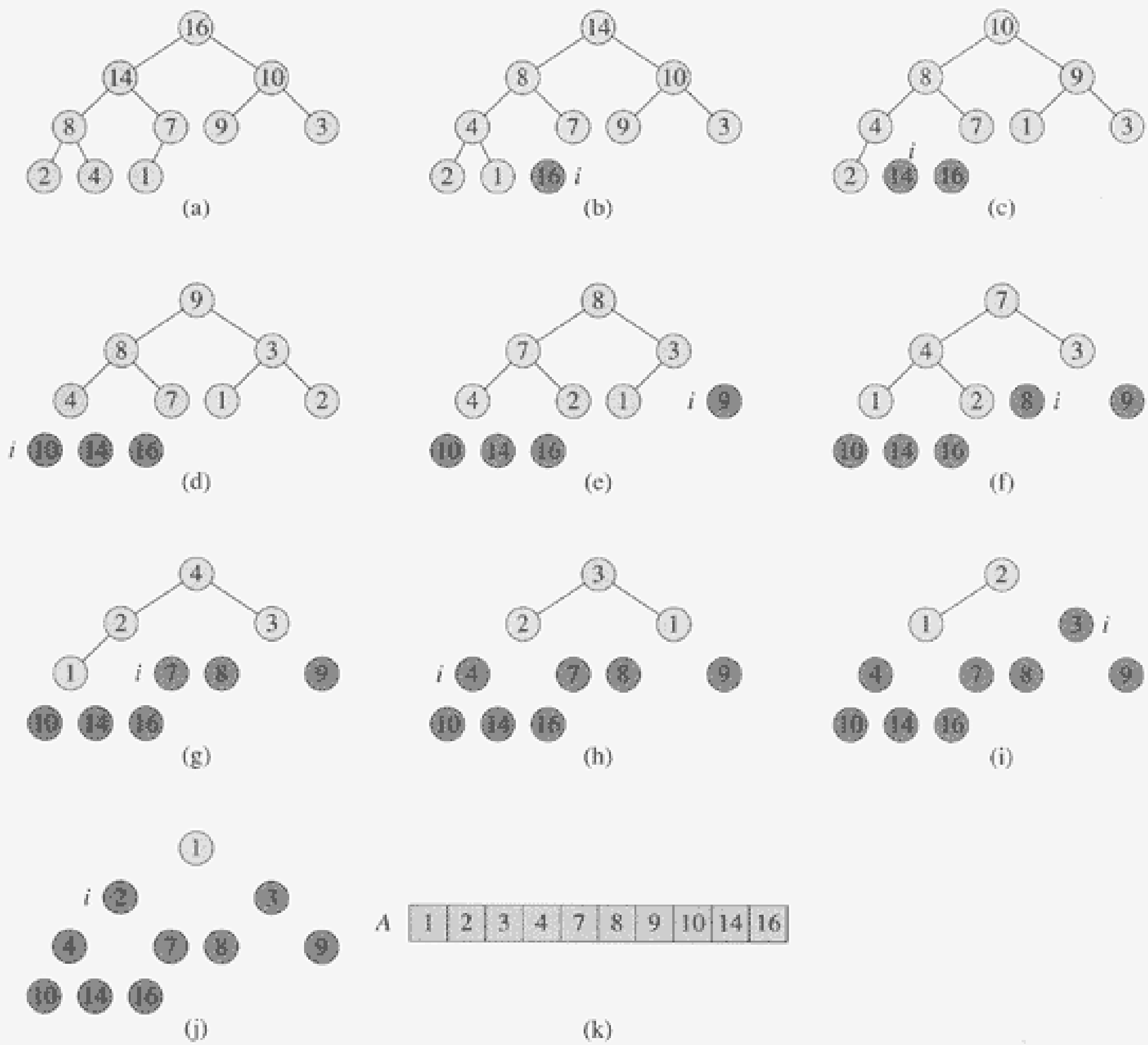

Figure 6.4 The operation of HEAPSORT. **(a)** The max-heap data structure just after it has been built by BUILD-MAX-HEAP. **(b)–(j)** The max-heap just after each call of MAX-HEAPIFY in line 5. The value of *i* at that time is shown. Only lightly shaded nodes remain in the heap. **(k)** The resulting sorted array *A*.

```
HEAP-MAXIMUM(A)
1  return A[1]
```

```
HEAP-EXTRACT-MAX(A)
if heap-size[A] < 1
    then error "heap underflow"
max ← A[1]
A[1] ← A[heap-size[A]]
heap-size[A] ← heap-size[A] − 1
MAX-HEAPIFY(A, 1)
return max
```

```
HEAP-INCREASE-KEY(A, i, key)
if key < A[i]
    then error "new key is smaller than current key"
A[i] ← key
while i > 1 and A[PARENT(i)] < A[i]
    do exchange A[i] ↔ A[PARENT(i)]
        i ← PARENT(i)
```

```
MAX-HEAP-INSERT(A, key)
1   heap-size[A] ← heap-size[A] + 1
2   A[heap-size[A]] ← −∞
3   HEAP-INCREASE-KEY(A, heap-size[A], key)
```

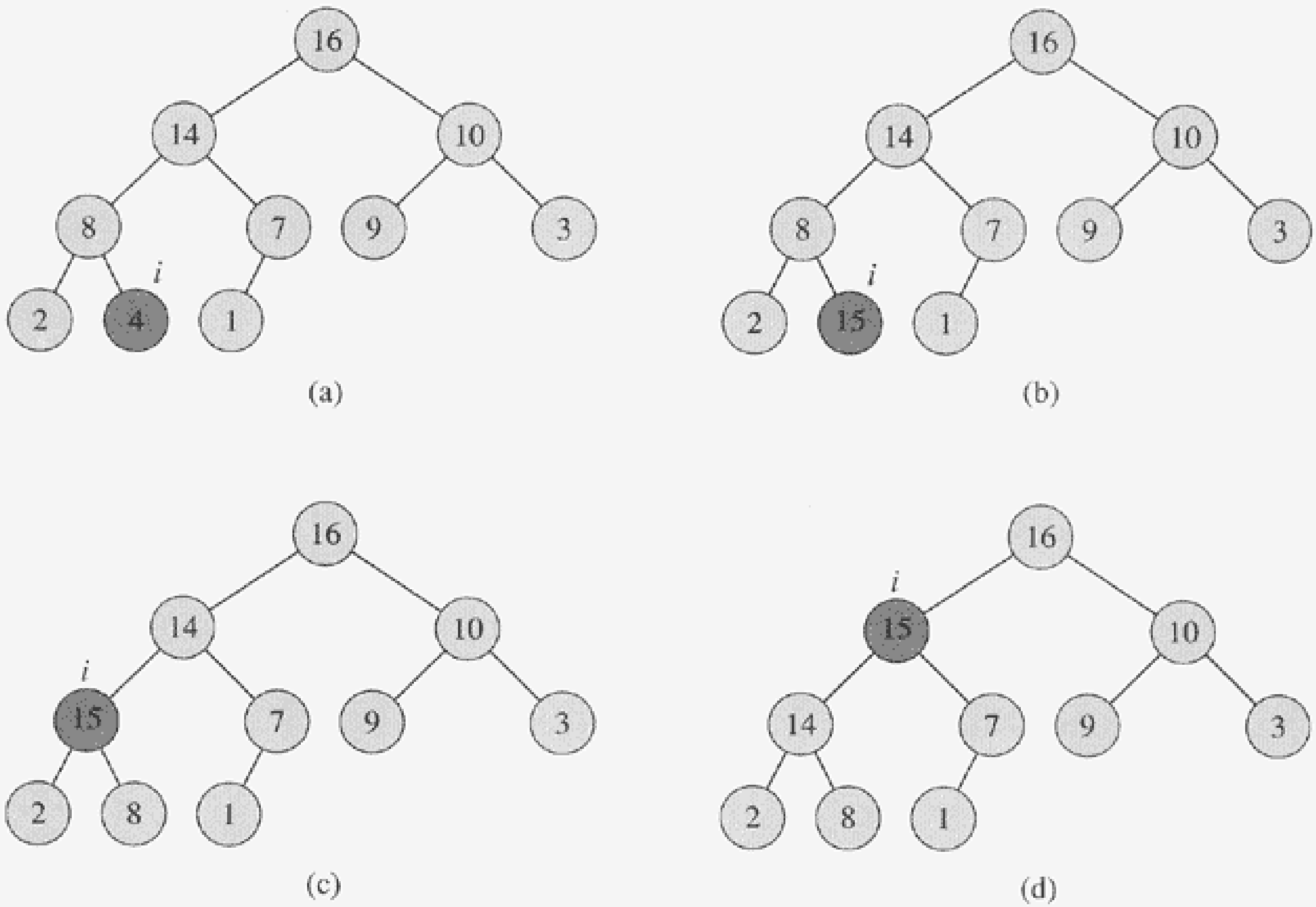

Figure 6.5 The operation of HEAP-INCREASE-KEY. **(a)** The max-heap of Figure 6.4(a) with a node whose index is i heavily shaded. **(b)** This node has its key increased to 15. **(c)** After one iteration of the **while** loop of lines 4–6, the node and its parent have exchanged keys, and the index i moves up to the parent. **(d)** The max-heap after one more iteration of the **while** loop. At this point, $A[\text{PARENT}(i)] \geq A[i]$. The max-heap property now holds and the procedure terminates.

```
BUILD-MAX-HEAP(A)
heap-size[A] ← length[A]
for i ← ⌊length[A]/2⌋ downto 1
      do MAX-HEAPIFY(A, i)
```