Test of Significance and related terms

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Outline

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5. Power of the test
6. Level of Significance
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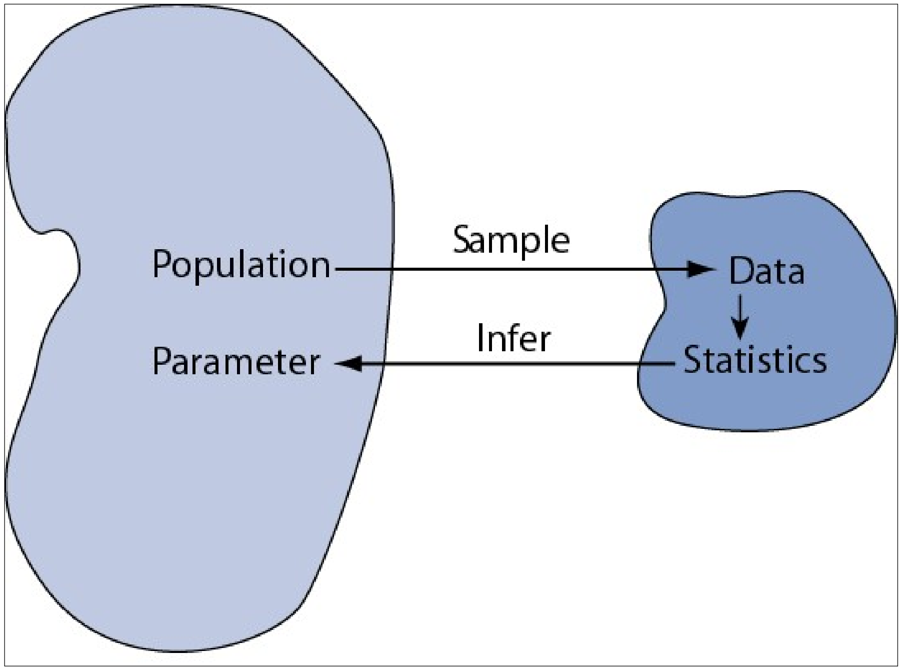
Preliminaries

- Population ≡ all possible values
- Sample ≡ a portion of the population
- Statistical inference ≡ generalizing from a sample to a population with calculated degree of certainty
- Two forms of statistical inference
  - Hypothesis testing
  - Estimation
- Parameter ≡ a characteristic of population, e.g., population mean μ
- Statistic ≡ calculated from data in the sample, e.g., sample mean ( )

Preliminaries..



Sampling Distributions of a Mean

- The **sampling distributions of a mean (SDM)** describes the behavior of a sampling mean

$$\bar{x} \sim N\left(\mu, SE_{\bar{x}}\right)$$

$$\text{where } SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

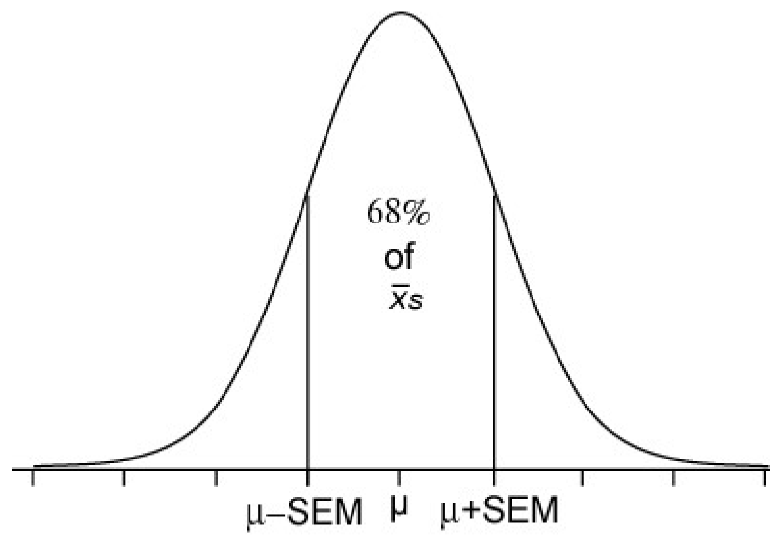


Fig. sdm&se68%.ai

Test of Significance

The test which is done for testing the research hypothesis against the null hypothesis.

OR

Test of significance is a formal procedure for comparing observed data with a claim (also called a hypothesis) whose truth we want to assess.

Hypothesis Testing

- Is also called *significance testing*
- Tests a claim about a parameter using evidence (data in a sample
- The technique is introduced by considering a one-sample z test
- The procedure is broken into four steps
- *Each* element of the procedure must be understood

Hypothesis Testing Steps

A. Null and alternative hypotheses
B. Test statistic
C. P-value and interpretation
D. Significance level (optional)

Null and Alternative Hypotheses

- Convert the research question to null and alternative hypotheses
- The **null hypothesis (*H_0*)** is a claim of "no difference in the population"
- The **alternative hypothesis (*H_a*)** claims "H_0 is false"
- Collect data and seek evidence against H_0 as a way of bolstering H_a (deduction)

Illustrative Example: “Body Weight”

The problem: In the 1970s, 20–29 year old men in the U.S. had a mean μ body weight of 170 pounds. Standard deviation σ was 40 pounds. We test whether mean body weight in the population now differs.

- **Null hypothesis** $H_{0:}$ $\mu = 170$ (“no difference”)
- The **alternative hypothesis** can be either $H_{a:}$ $\mu > 170$ (**one-sided test**) or
  $H_{a:}$ $\mu \neq 170$ (**two-sided test**)

Test Statistic

This is an example of a one-sample test of a mean when σ is known. Use this statistic to test the problem:

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}}$$

where $\mu_0 \equiv$ population mean assuming H_0 is true

$$\text{and } SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Illustrative Example: *z* statistic

- For the illustrative example, μ_0 = 170
- We know σ = 40
- Take an SRS of *n* = 64. Therefore

$$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{40}{\sqrt{64}} = 5$$

- If we found a sample mean of 173, then

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{173 - 170}{5} = 0.60$$

Illustrative Example: *z* statistic

If we found a sample mean of 185, then

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{185 - 170}{5} = 3.00$$

Errors in Hypothesis Testing

	Truth	
Decision	**Null hypothesis**	**Alternative hypothesis**
Do not reject null	OK	TYPE II ERROR
Reject null	TYPE I ERROR	OK

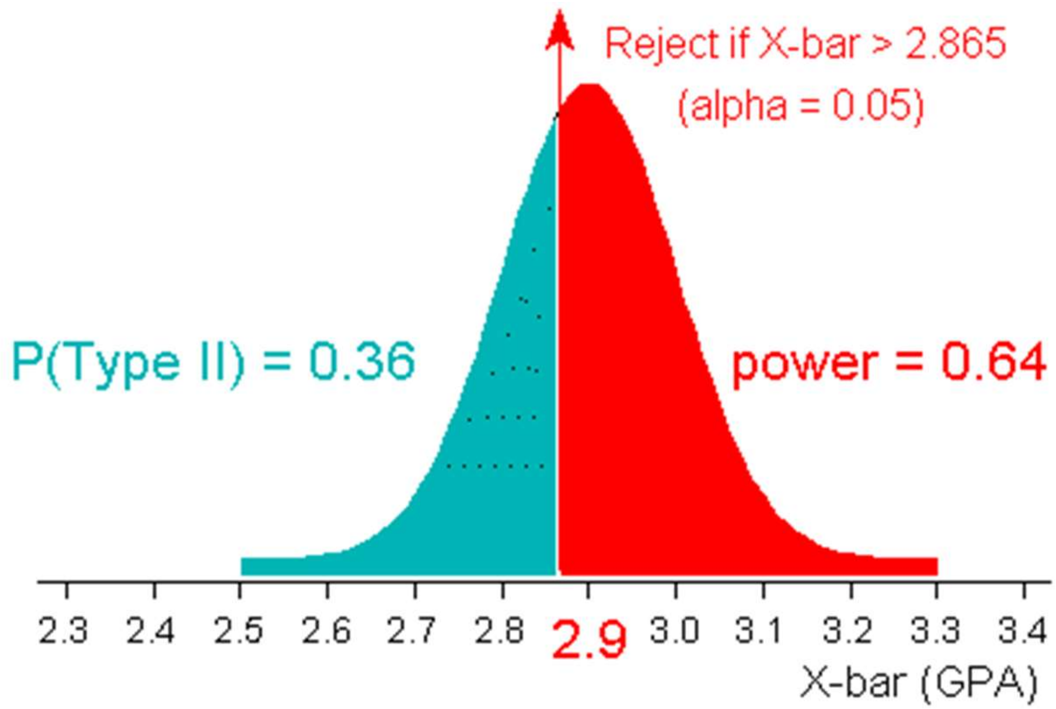
Definitions: Types of Errors

- **Type I error**: The null hypothesis is rejected when it is true.
- **Type II error**: The null hypothesis is not rejected when it is false.
- There is always a chance of making one of these errors. We'll want to minimize the chance of doing so!

Type II Error and Power

- "**Power**" of a test is the probability of rejecting null when alternative is true.
- **"Power" = 1 - P(Type II error)**
- To minimize the P(Type II error), we equivalently want to maximize power.
- But power depends on the value under the alternative hypothesis ...

Type II Error and Power



Power of a z test

$$1-\beta = \Phi\left(-z_{1-\frac{\alpha}{2}} + \frac{|\mu_0 - \mu_a|\sqrt{n}}{\sigma} \right)$$

where
$\Phi(z)$ represent the cumulative probability of Standard Normal Z
μ_0 represent the population mean under the null hypothesis
μ_a represents the population mean under the alternative hypothesis

Calculating Power: Example

A study of n = 16 retains H_0: μ = 170 at α = 0.05 (two-sided); σ is 40. What was the power of test's conditions to identify a population mean of 190?

$$1-\beta = \Phi\left(-z_{1-\frac{\alpha}{2}} + \frac{|\mu_0 - \mu_a|\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(-1.96 + \frac{|170-190|\sqrt{16}}{40}\right)$$

$$= \Phi(0.04)$$

$$= 0.5160$$

Level of Significance

- Probability of making type-I error.
- … by making significance level α small.
- Common values are α = 0.01, 0.05, or 0.10.
- “How small” depends on seriousness of Type I error.
- Decision is not a statistical one but a practical one.

P-value

- The *P*-value answer the question: What is the probability of the observed test statistic or one more extreme **when H_0 *is true*?**
- This corresponds to the tail of the Standard Normal distribution beyond the $z_{stat.}$
- Convert *z* statistics to *P*-value :

  For H_a: $\mu > \mu_0 \Rightarrow P = \Pr(Z > z_{stat})$ = right-tail beyond z_{stat}

  For H_a: $\mu < \mu_0 \Rightarrow P = \Pr(Z < z_{stat})$ = left tail beyond z_{stat}

  For H_a: $\mu \neq \mu_0 \Rightarrow P = 2 \times$ one-tailed *P*-value