TESTING HYPOTHESIS

Hypothesis:- A statistical hypothesis is an assertion or statement about a population or equivalently about the probability distribution characterizing a population, which we want to verify on the basis of information contained in a sample.

Examples:-

(1) A physician may hypothesize that the recommended drug is effective in 90 percent cases.
(2) A sewing machine company claims that their new machine is superior to the one available in the market.

Test of a statistical hypothesis:- A test of a statistical hypothesis is a two-action decision problem after the experimental sample values have been obtained, the two-actions being the acceptance or rejection of the hypothesis under consideration.

Null hypothesis:- Null means the possible rejection of the hypothesis. Null hypothesis is a statement, which tells us that no difference exists between the parameter and the statistic being compared to it. Null hypothesis is always denoted by H_0.

Example:- There is no difference in the population between the rates of prevalence of malnutrition between the male and female children.

Alternative hypothesis:- The alternative hypothesis is the logical opposite of the null hypothesis. Alternative hypothesis is usually denoted by H_1 or H_a.

Example:- There exists significant difference in the population between the rates of prevalence of malnutrition between the male and female children.

H_0: There is no association between level of education and knowledge of child nutrition among women.

H_1: The level of education and knowledge of child nutrition among women are associated.

Simple hypothesis:- If a hypothesis completely specifies the distribution of a population, it is called a simple hypothesis. Suppose, for example, that a coin is tossed 30 times ($n = 30$) to determine whether the coin is an ideal one. Then, the hypothesis $H: \; p = 0.50$ is a simple hypothesis since it completely specifies the population distribution.

Composite hypothesis:- If a hypothesis does not completely specify the population distribution completely, it is called a composite hypothesis. In the above coin-tossing example, if we do not specify that $n = 30$, then the hypothesis $H: \; p = 0.50$ would be a composite hypothesis.

One-tailed test:- A test of any statistical hypothesis where the alternative is one-sided such as

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$

or perhaps, $H_0 : \mu = \mu_0$

$H_1 : \mu < \mu_0$

is called a one-sided test.

Two-tailed test:- A test of any statistical hypothesis where the alternative is two-sided such as

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

is called a two-tailed test.

Type I error:- The error of rejecting H_0 (accepting H_1) when H_0 is true is called type I error. The probability of type I error is denoted by α and it is called the level of significance.

Type II error:- The error of accepting H_0 when H_0 is false (H_1 is true) is called type II error. The probability of type II error is denoted by β.

	Status of Ho	
	Ho true	Ho false
Accept Ho	Correct decision	Type II error
Reject Ho	Type I error	Correct decision

Power of the test:- $1-\beta$, that is the probability of rejecting H_0 when H_0 is false (H_1 is true) is called the power of the test hypothesis H_0 against the alternative hypothesis H_1.

Critical region or rejection region:- A region of rejection is a set of possible values of the sample statistic, which provides evidence to contradict the null hypothesis and leads to a decision to reject the null hypothesis.

Acceptance region:- A region of acceptance is a set of possible values of the sample statistic, which provides evidence to support the null hypothesis and lead to a decision to accept it.

Test statistic:- The statistic used to provide evidence about the null hypothesis is called the test statistic.

<u>Five step procedures for testing a hypothesis:-</u>

<u>Step1:Set Up null hypothesis(Ho) and the alternate hypothesis(H1):</u>

Null hypothesis:

A statement about the value of a population parameter.

Alternate hypothesis:

A statement that is accepted if the sample data provide sufficient evidence that null hypothesis is false.

The following example will help clarify what is meant by the null hypothesis and the alternate hypothesis. A recent article indicated the mean age of US commercial aircraft is 15 years. To conduct a statistical test regarding this estimate, the first step is to determine the null and the alternate hypothesis.

The null hypothesis represents the current or reported condition. it is written by Ho: μ =15

The alternate hypothesis is that the statement is not true, that is writtenbyH1: $\mu \neq 15$

It is important to remember that no matter how the problem is stated; the null hypothesis will always contain the equal sign. We turn to the alternate hypothesis only if the data suggests the null hypothesis is untrue.

<u>Step2: select level of significance:</u>

Level of significance is the probability of rejecting the null hypothesis when it is true. The level of significance is denoted by the Greek letter alpha. It is also sometimes called the level of risk.There is no one level of significance that is applied to all tests. Traditionally .o5 level is selected for the consumer research project, .01 for quality assurance and .10 for political polling.

<u>Step 3: select the test statistic:</u>

A value, determined from sample information, used to determine whether to reject the null hypothesis. Every test statistic follows a particular distribution under null hypothesis. They usually follow Z, t, F and so on.

<u>Step 4: set decision rule:</u>

A decision rule is a statement of conditions under which the null hypothesis rejected or the conditions under which it is not rejected. In this step, we are to define acceptance and rejection region of Ho.

<u>Step 5: decision making and interpretation:</u>

The last step is to make decision on the basis of the calculated value of test statistic to find it we are to take random sample.

A test of the mean of a normal distribution against two-sided alternative: σ known:-

1. $H_0: \mu = \mu_0$, Vs $H_1: \mu \neq \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_{\alpha/2}$, or $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > Z_{\alpha/2}$.
5. Make decision.

Example:- The production manager of Circuits Unlimited has asked for your assistance in analyzing a production process. The process involves drilling holes whose diameters are normally distributed with mean 2 inches and population standard deviation 0.06 inches. A random sample of 9 measurements had a sample mean of 1.95 inches. Use a significance level of $\alpha = 0.05$ to determine if the observed sample mean is unusual and suggests that the drilling machine should be adjusted.

Solution:-

1. $H_0: \mu = 2$, Vs $H_1: \mu \neq 2$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_{\alpha/2}$, or $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > Z_{\alpha/2}$.
5. Now, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{1.95 - 2}{0.06/\sqrt{9}} = -2.50$.

Comment:- For a 5% level test $\alpha = 0.05$ and $Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$. Thus since -2.50 is less than -1.96, we may reject the null hypothesis and conclude that the drilling machine requires adjustment

Significance level	**Two-tailed test**	**(Right- tailed)**	**(Left-tailed)**
0.05	± 1.96	1.65	-1.65
0.01	± 2.28	2.33	-2.33

A test of the mean of a normal distribution against right-sided alternative: σ known:-

1. $H_0: \mu = \mu_0$, Vs $H_1: \mu > \mu_0$.
2. Level of significance $= \alpha$.

3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.

4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > Z_\alpha$.

5. Make decision.

Example:- The production manager of Northern Windows Inc. has asked you to evaluate a proposed new procedure for producing its Regal Line of double-hung windows. The present process has a mean production of 80 units per hour with a population standard deviation of $\sigma = 8$. The manager indicates that she does not want to change to a new procedure unless there is strong evidence that the mean production level is higher with the new process. A random sample of 25 production hours was selected and the sample mean was 83 units per hour.

Solution:-

1. $H_0: \ \mu = 80$, Vs $H_1: \ \mu > 80$.
2. Let the level of significance $\alpha = 0.05$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > Z_\alpha$.
5. Now, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{83-80}{8/\sqrt{25}} = 1.875$.

Comment:- For a 5% level test $\alpha = 0.05$ and $Z_\alpha = Z_{0.05} = 1.645$. Thus since 1.875 is greater than 1.645 we would reject the null hypothesis and conclude that there was strong evidence to support the conclusion that the new process resulted in higher productivity.

A test of the mean of a normal distribution against left-sided alternative: σ known:-

1. $H_0: \ \mu = \mu_0$, Vs $H_1: \ \mu < \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_\alpha$.
5. Make decision.

Example:- An internet server claims that its users spend on the average 20 hours per week with a standard deviation of 2.5 hours on the information superhighway. To determine whether this is an overestimate, a competitor conducted a sample survey of 15 customers and found that the average time spent online was 21.8 hours per week. Do the data provide sufficient evidence to indicate that the average hours of use are less than that claimed by the first internet? Test at 5% level.

Solution:-

1. $H_0:\ \mu = 20$, Vs $H_1:\ \mu < 20$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_\alpha$.
5. Now, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{21.8 - 20}{2.5/\sqrt{15}} = 2.79$.

Comment:- For a 5% level test $\alpha = 0.05$ and $Z_\alpha = Z_{0.05} = 1.645$. Thus since computed Z value is greater than the critical value $-Z_\alpha = -1.645$, we cannot reject the null hypothesis and conclude that there is sufficient evidence to agree with the claim of the first internet server.

A test of the mean of a normal distribution against two-sided alternative: σ unknown and sample size large $(n \geq 30)$:-

1. $H_0:\ \mu = \mu_0$, Vs $H_1:\ \mu \neq \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} < -Z_{\alpha/2}$, or $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} > Z_{\alpha/2}$.
5. Make decision.

Example:- Given the following hypothesis:

$$H_0:\ \mu = 400,\ \text{Vs}\ H_1:\ \mu \neq 400$$

For a random sample of 32 observations, the sample mean was 407 and the standard deviation 6. Using the 0.05 significance level, what is your decision regarding the null hypothesis?

Solution:-

1. $H_0:\ \mu = 400$, Vs $H_1:\ \mu \neq 400$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} < -Z_{\alpha/2}$, or $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} > Z_{\alpha/2}$.

5. Now, $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{407 - 400}{6/\sqrt{32}} = 6.60$.

Comment:- For a 5% level test $\alpha = 0.05$ and $Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$. Thus since 6.60 is greater than 1.96, we may reject the null hypothesis.

A test of the mean of a normal distribution against right-sided alternative: σ unknown and sample size large $(n \geq 30)$:-

1. $H_0:\ \mu = \mu_0$, Vs $H_1:\ \mu > \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} > Z_\alpha$.
5. Make decision.

Example:- The Thompson's Discount Appliance Store issues its own credit card. The credit manager wants to find whether the mean monthly unpaid balance is more than $400. The level of significance is set at 0.05. A random check of 172 unpaid balance revealed the sample mean is $407 and the standard deviation of the sample is $38. Should the credit manager conclude the population mean is greater than $400?

Solution:-

1. $H_0:\ \mu = 400$, Vs $H_1:\ \mu > 400$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} > Z_\alpha$.
5. Now, $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{407 - 400}{38/\sqrt{172}} = 2.42$.

Comment:- For a 5% level test $\alpha = 0.05$ and $Z_\alpha = Z_{0.05} = 1.645$. Thus since 2.42 is greater than 1.645, we would reject the null hypothesis. The credit manager can conclude the mean unpaid balance is greater than $400.

A test of the mean of a normal distribution against left-sided alternative: σ unknown and sample size large $(n \geq 30)$:-

1. $H_0: \mu = \mu_0$, Vs $H_1: \mu < \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} < -Z_\alpha$.
5. Make decision.

Example:- The weight of a sample of 75 capsules are to be used to examine whether there is sufficient empirical evidence that the capsules of this type have a mean weight less than 10g. The sample gave a mean weight of 9.77g with a standard deviation 0.50. Perform an appropriate statistical test to provide evidence in support of the claim at 5 percent level of significance.

Solution:-

1. $H_0: \mu = 10$, Vs $H_1: \mu < 10$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim N(0,1)$.
4. Reject H_0, if $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} < -Z_\alpha$.
5. Now, $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{9.77 - 10}{0.5/\sqrt{75}} = -3.98$.

Comment:- For a 5% level test $\alpha = 0.05$ and $Z_\alpha = Z_{0.05} = 1.645$. Thus since -3.98 is less than -1.645, we may reject the null hypothesis and conclude that the mean weight is less than 10g.

A test of the mean of a normal distribution against two-sided alternative: σ unknown and sample size small $(n < 30)$:-

1. $H_0: \mu = \mu_0$, Vs $H_1: \mu \neq \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim t_{(n-1)}$.
4. Reject H_0, if $t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} < -t_{\alpha/2,(n-1)}$, or $t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} > t_{\alpha/2,(n-1)}$.
5. Make decision.

Example:- The mean length of a small counterbalance bar is 43 millimeters. The production supervisor is concerned that the adjustments of the machine producing the bars have charged. He asks the Engineering Department to investigate. Engineering selects a random sample of 12 bars and measures each. The results are reported below in millimeters:

42, 39, 42, 45, 43, 40, 39, 41, 40, 42, 43, 42

Is it reasonable to conclude that there has been a change in the mean length of the bars? Use the 0.05 significance level.

Solution:-

1. $H_0:\ \mu = 43$, Vs $H_1:\ \mu \neq 43$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{(n-1)}$.
4. Reject H_0, if $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} < -t_{\alpha/2,(n-1)}$, or $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} > t_{\alpha/2,(n-1)}$.
5. Now, $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} = \dfrac{41.5 - 43}{1.784/\sqrt{12}} = -2.91$.

Comment:- For a 5% level test $\alpha = 0.05$ and $t_{\alpha/2,(n-1)} = t_{0.05/2,(12-1)} = t_{0.05/2,(11)} = 2.201$. Thus since -2.91 is less than -2.201, we may reject the null hypothesis and conclude that the population mean is not 43 millimeters. The machine is out of control and needs adjustment.

A test of the mean of a normal distribution against right-sided alternative: σ unknown and sample size small $(n < 30)$:-

1. $H_0:\ \mu = \mu_0$, Vs $H_1:\ \mu > \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{(n-1)}$.
4. Reject H_0, if $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} > t_{\alpha,(n-1)}$.
5. Make decision.

Example:- The average IQ of university women in Bangladesh is suspected to be more than the average 110 for all students. A random sample of 24 women yielded an average IQ of 115.5 and standard deviation of 20. Can you conclude that the average score of the women in the population is really more than 110? Test this at 5 percent level of significance.

Solution:-

1. $H_0:\ \mu = 110$, Vs $H_1:\ \mu > 110$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{(n-1)}$.

4. Reject H_0, if $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} > t_{\alpha,(n-1)}$.

5. Now, $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} = \dfrac{115.5 - 110}{20/\sqrt{24}} = 1.347$.

Comment:- For a 5% level test $\alpha = 0.05$ and $t_{\alpha,(n-1)} = t_{0.05,(24-1)} = t_{0.05,(23)} = 1.714$. Thus since 1.347 is less than 1.714, we may not reject the null hypothesis and conclude that the mean score of the women in the population is not really more than 110.

A test of the mean of a normal distribution against left-sided alternative: σ unknown and sample size small $(n < 30)$:-

1. $H_0:\ \mu = \mu_0$, Vs $H_1:\ \mu < \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim t_{(n-1)}$.
4. Reject H_0, if $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} < -t_{\alpha,(n-1)}$.
5. Make decision.

Example:- The average waiting time in a bank counter to cash a cheque for all customers has been 50 minutes. A new service-providing procedure using modern computer facilities is being tried. If a random sample of 12 customers had an average waiting time for services is 42 minutes with a standard deviation of 11.9 minutes under the new system, test the hypothesis that the population mean is now less than 50, using a significance level of 5 percent.

Solution:-

1. $H_0:\ \mu = 50$, Vs $H_1:\ \mu < 50$.
2. Level of significance, $\alpha = 0.05$..
3. Test statistic, $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim t_{(n-1)}$.
4. Reject H_0, if $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} < -t_{\alpha,(n-1)}$.
5. Now, $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} = \dfrac{42 - 50}{11.9/\sqrt{12}} = -2.33$.

Comment:- For a 5% level test $\alpha = 0.05$ and $t_{\alpha,(n-1)} = t_{0.05,(12-1)} = t_{0.05,(11)} = 1.796$. Thus since -2.33 is less than -1.796, we may reject the null hypothesis. This implies that the true mean is likely to be less than 50 minutes.