String Matching

- *Using Finite Automata*

Example (I)

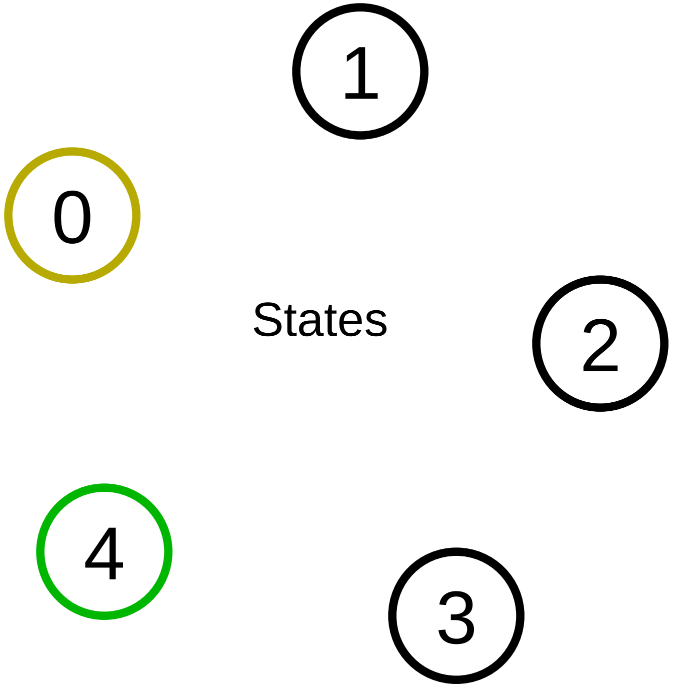
Q is a finite set of states

$q_0 \in \mathbf{Q}$ is the start state

Q is a set of accepting sates

Σ: input alphabet

$\delta: Q \times \Sigma \rightarrow Q$: transition function



input: | a | b | a | b | b | a | b | b | a | a |

Example (II)

Q is a finite set of states

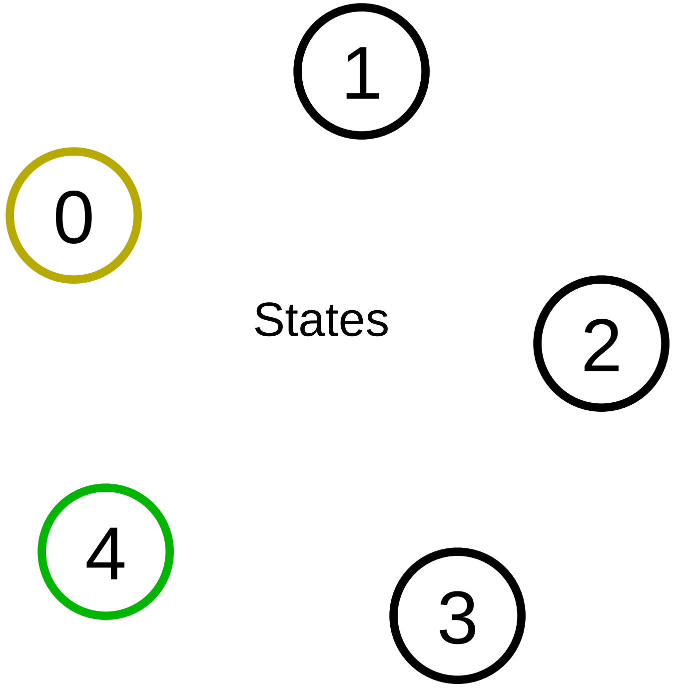
$q_0 \in \mathbf{Q}$ is the start state

Q is a set of accepting sates

Σ: input alphabet

δ: $Q \times \Sigma \rightarrow Q$: transition function

state \ input	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2



Example (III)

Q is a finite set of states

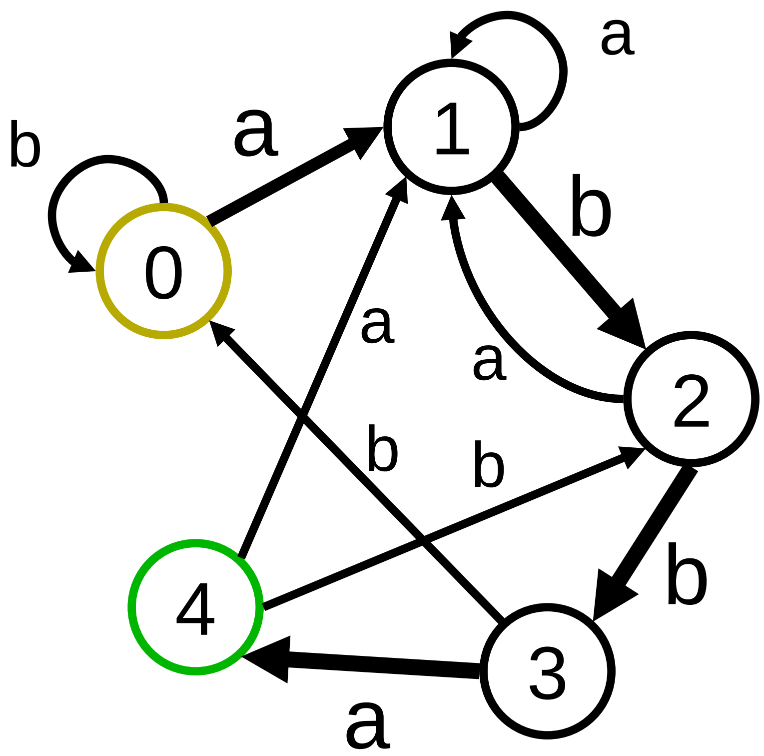
$q_0 \in$ **Q** is the start state

Q is a set of accepting sates

Σ: input alphabet

$\delta: Q \times \Sigma \rightarrow Q$: transition function

state \ input	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2



Example (IV)

Q is a finite set of states

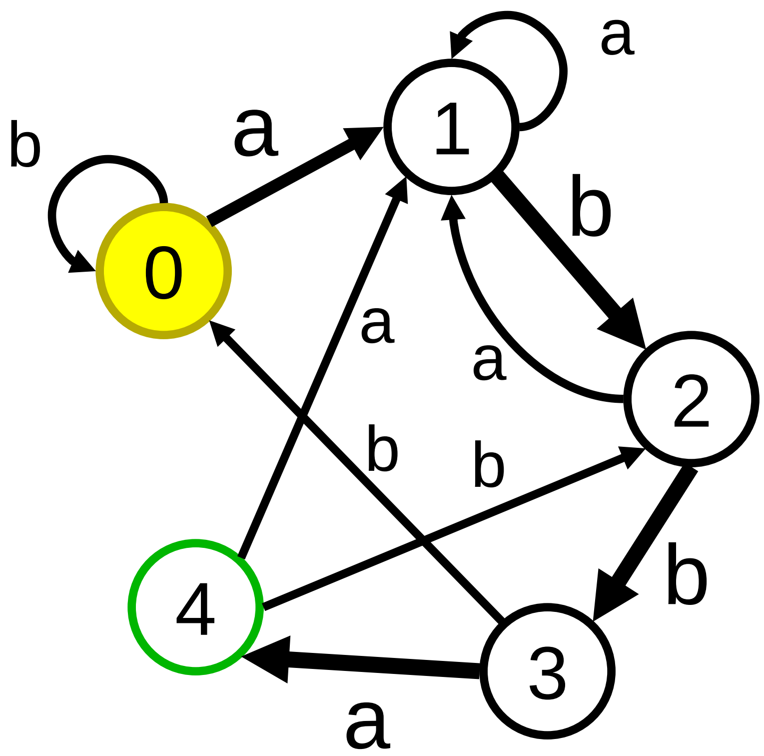
$q_0 \in \mathbf{Q}$ is the start state

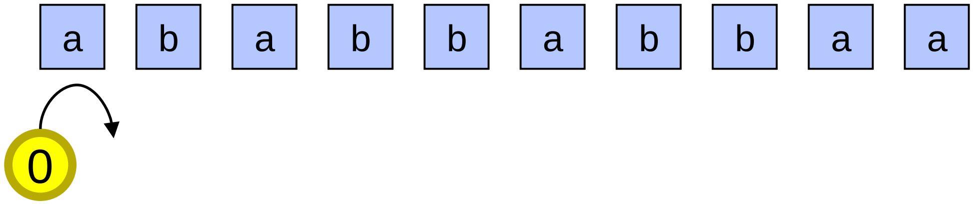
Q is a set of accepting sates

Σ: input alphabet

$\delta: Q \times \Sigma \rightarrow Q$: transition function

state \ input	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2





Example (V)

Q is a finite set of states

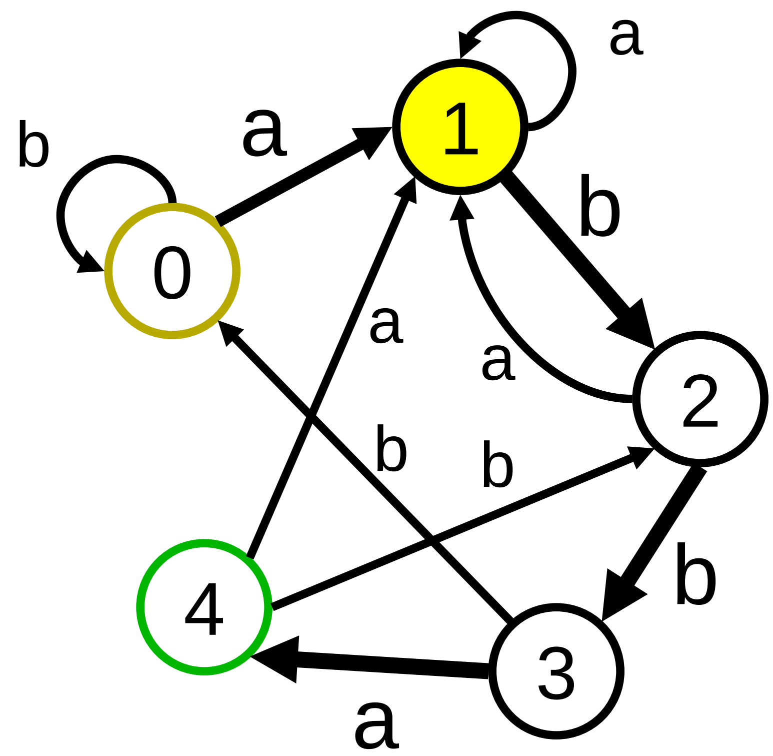
$q_0 \in \mathbf{Q}$ is the start state

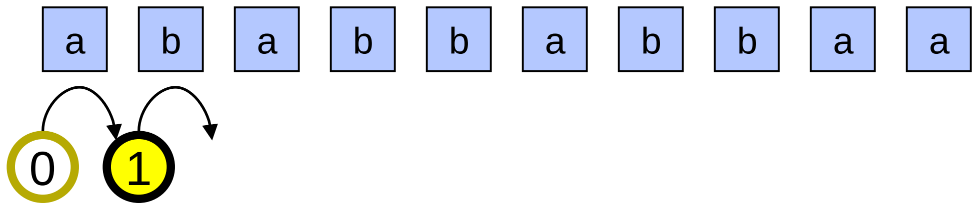
Q is a set of accepting sates

Σ: input alphabet

$\delta: Q \times \Sigma \rightarrow Q$: transition function

state \ input	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2





Example (VI)

Q is a finite set of states

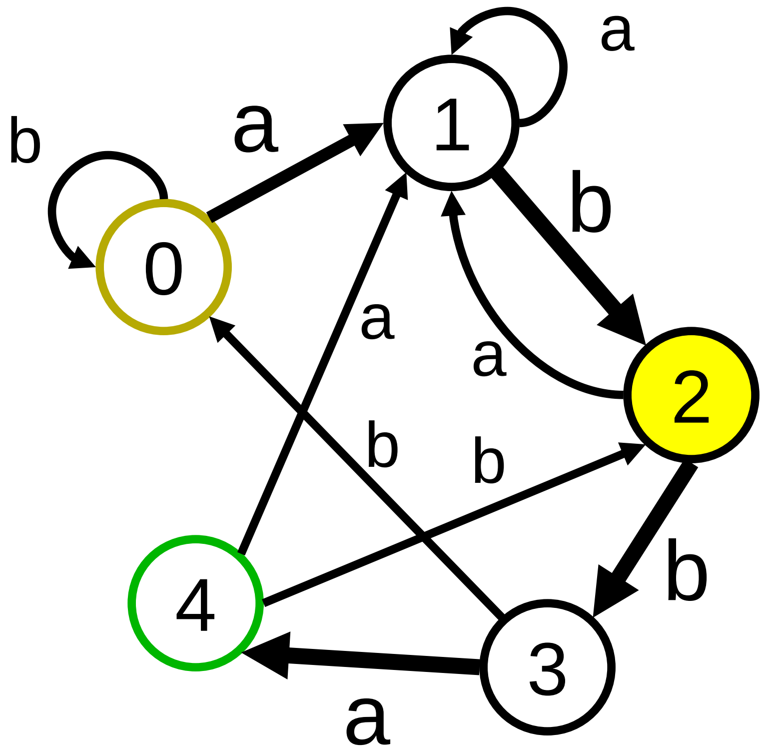
$q_0 \in$ **Q** is the start state

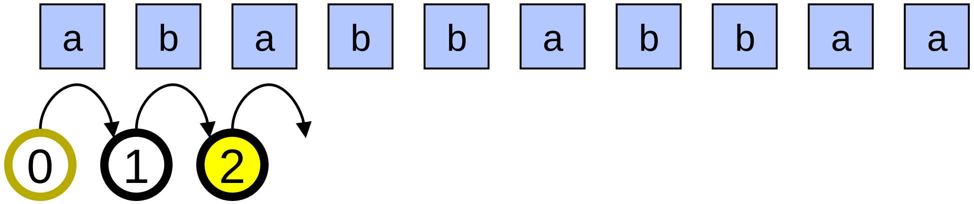
Q is a set of accepting sates

Σ: input alphabet

$\delta: Q \times \Sigma \rightarrow Q$: transition function

state \ input	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2





Example (VII)

Q is a finite set of states

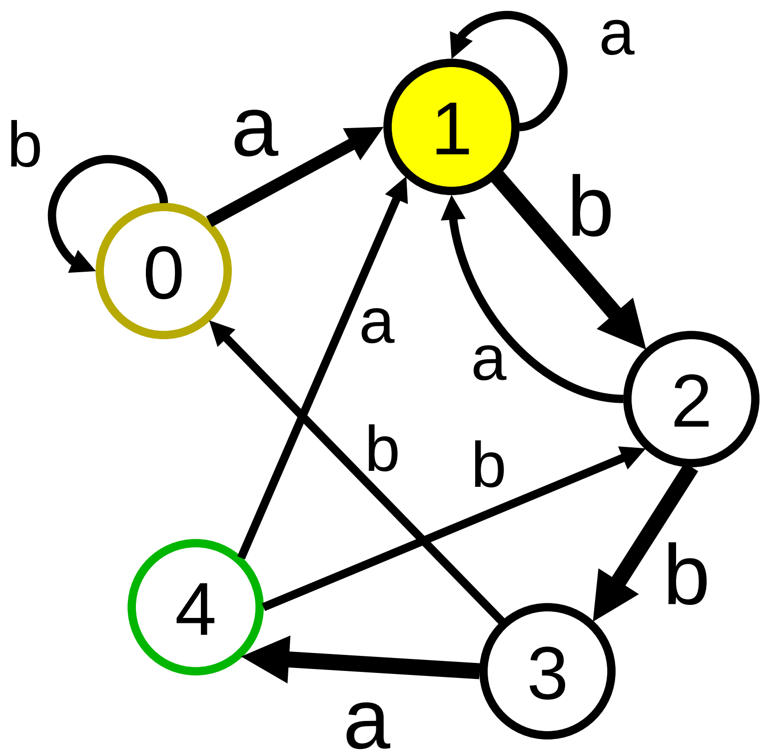
$q_0 \in \mathbf{Q}$ is the start state

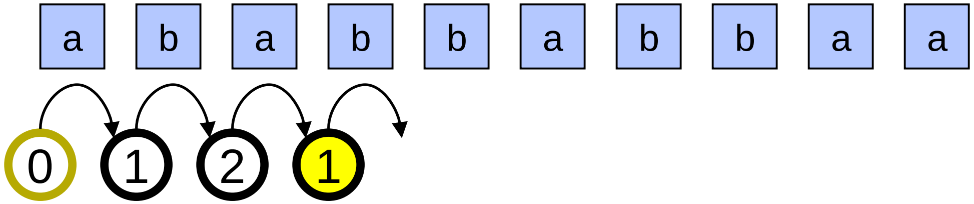
Q is a set of accepting sates

Σ: input alphabet

$\delta: Q \times \Sigma \rightarrow Q$: transition function

state \ input	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2





Example (VIII)

Q is a finite set of states

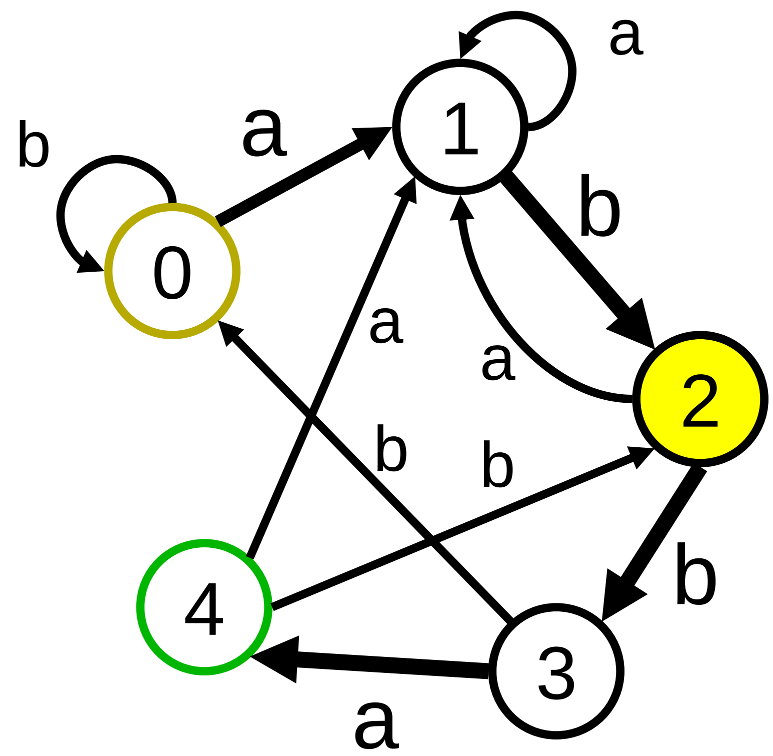
$q_0 \in \mathbf{Q}$ is the start state

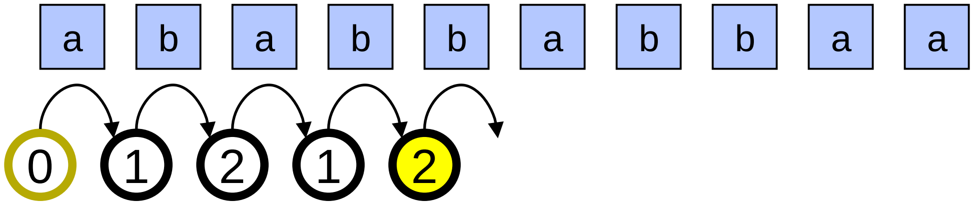
Q is a set of accepting sates

Σ: input alphabet

δ: $Q \times \Sigma \rightarrow Q$: transition function

input / state	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2





Example (IX)

Q is a finite set of states

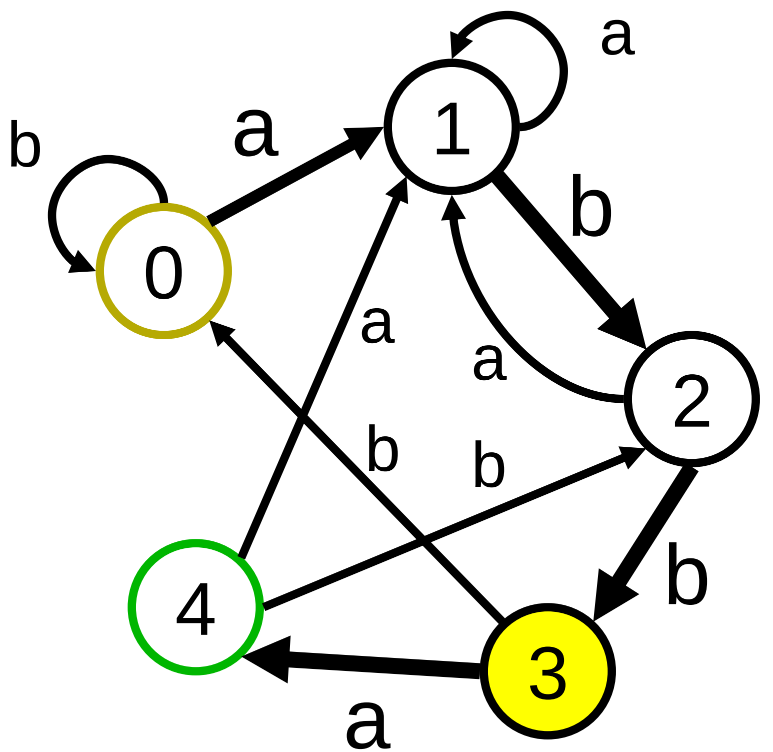
$q_0 \in$ **Q** is the start state

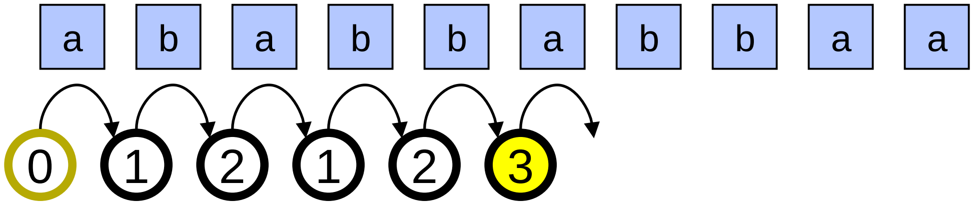
Q is a set of accepting sates

Σ: input alphabet

δ: $Q \times \Sigma \rightarrow Q$: transition function

state \ input	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	1	2





Example (X)

Q is a finite set of states

$q_0 \in \mathbf{Q}$ is the start state

Q is a set of accepting sates

Σ: input alphabet

$\delta: Q \times \Sigma \rightarrow Q$: transition function

input / state	a	b
0	1	0
1	1	2
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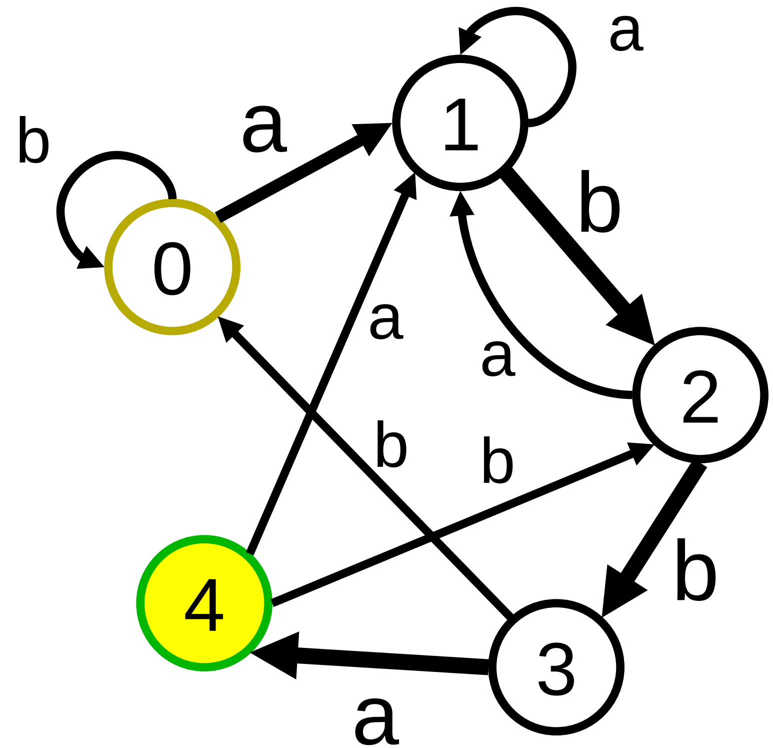
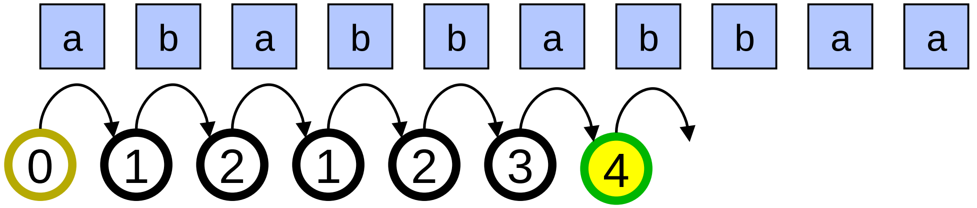
4	1	2





Example (XI)

Q is a finite set of states

$q_0 \in \mathbf{Q}$ is the start state

Q is a set of accepting sates

Σ: input alphabet

δ: $Q \times \Sigma \rightarrow Q$: transition function

input / state	a	b
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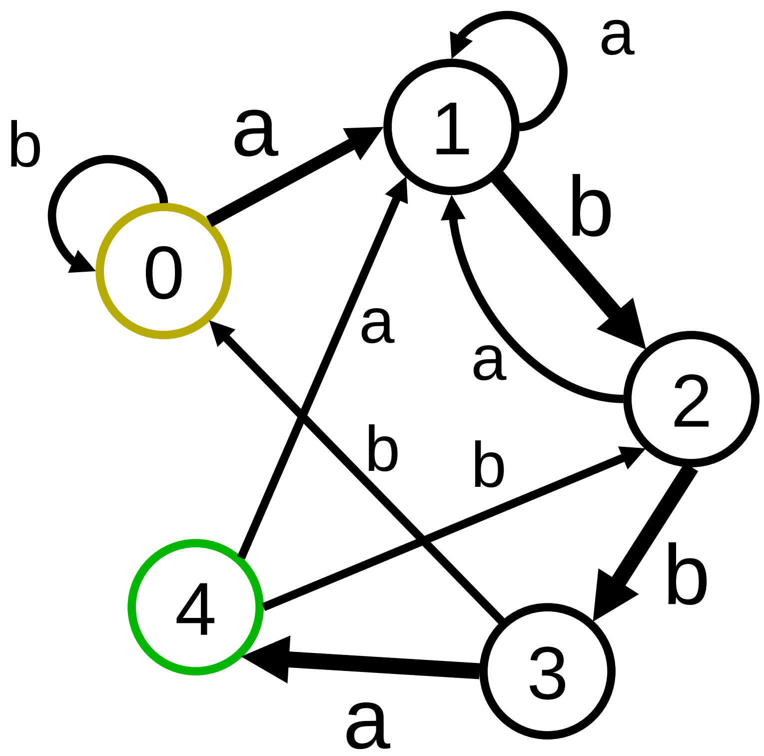

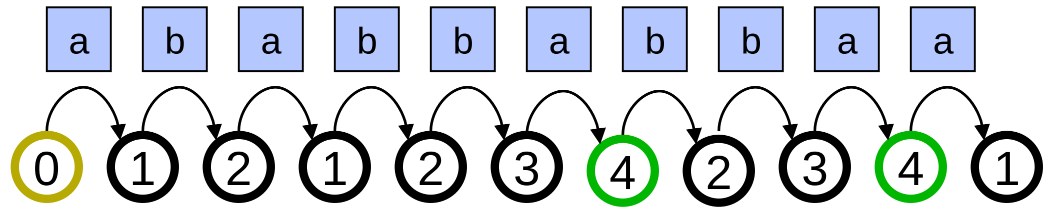

Finite-Automaton-Matcher

- **The example automaton accepts at the end of occurrences of the pattern abba**
- **For every pattern of length m there exists an automaton with m+1 states that solves the pattern matching problem with the following algorithm:**

Finite-Automaton-Matcher(T,δ,P)

1. **n $\leftarrow$ length(T)**
2. **q $\leftarrow$ 0**
3. **for i $\leftarrow$ 1 to n do**
4. **q $\leftarrow$ δ(q,T[i])**
5. **if q = m then**
6. **s $\leftarrow$ i - m**
7. **return “Pattern occurs with shift” s**

How to Compute the Transition Function?

- A string *u* is a **prefix** of string *v* if there exists a string *a* such that: $ua = v$
- A string *u* is a **suffix** of string *v* if there exists a string *a* such that: $au = v$
- Let P_k denote the first k letter string of P

Compute-Transition-Function(P, Σ)

1. **m $\leftarrow$ length(P)**
2. **for q $\leftarrow$ 0 to m do**
3. **for each character a $\in \Sigma$ do**
4. **k $\leftarrow$ 1+min(m,q+1)**
5. **repeat**
   k $\leftarrow$ k-1
6. **until P_k is a suffix of P_qa**
7. **δ(q,a) $\leftarrow$ k**

Example

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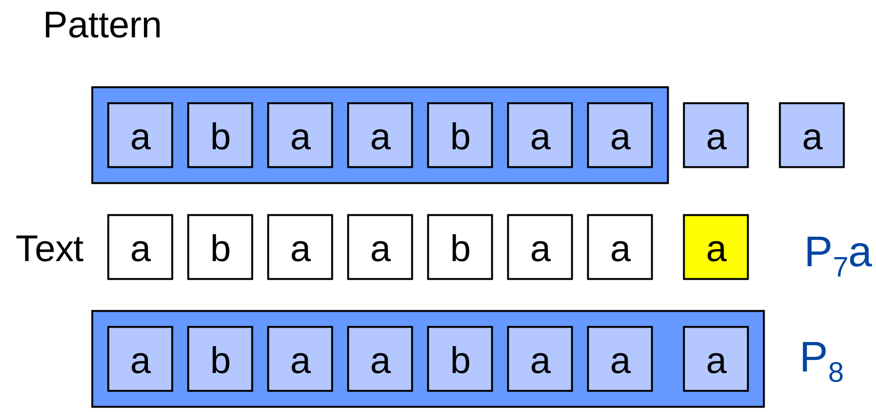
6. **until P_k is a suffix of P_qa**
7. **δ(q,a) $\leftarrow$ k**
8.



Example

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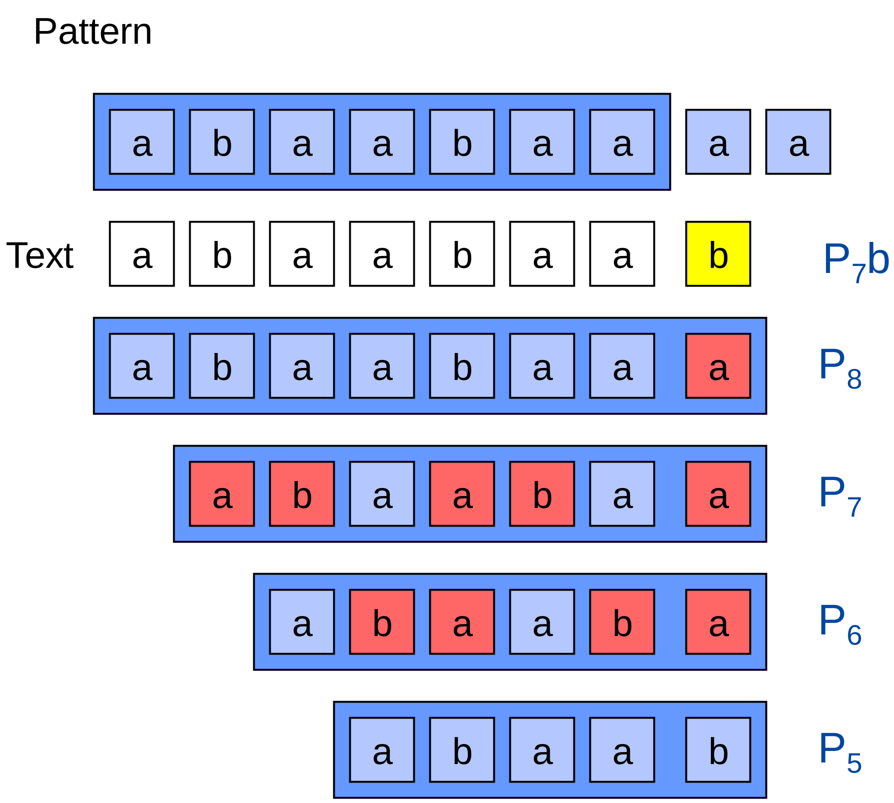
- Let P_k denote the first k letter string of P

Compute-Transition-Function(P, Σ)

```
1.  m ← length(P)
2.  for q ← 0 to m do
3.      for each character a ∈ Σ do
4.          k ← 1+min(m,q+1)
5.          repeat
                k ← k-1
6.          until P_k is a suffix of P_q a
7.          δ(q,a) ← k
8.
```



Running time of Compute Transition-Function

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- A string *u* is a **suffix** of string *v* if there exists a string *a* such that: *au* = *v*
- Let P_k denote the first k letter string of P

Compute-Transition-Function(P, Σ)

```
m ← length(P)
for q ← 0 to m do
      for each character a ∈ Σ do
            k ← 1+min(m,q+1)
            repeat
                  k ← k-1
            until P_k is a suffix of P_q a
            δ(q,a) ← k
```

Factor: m+1

Factor: |Σ|

Factor: m

Time for check of equality: m

Running time of procedure: $O(m^3 |\Sigma|)$