Lecture 2
Introduction & Fundamentals of Image Processing

Ref. *Digital Image Processing*

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Introduction

- What is Digital Image Processing?

Digital Image
— a two-dimensional function $f(x, y)$
x and y are spatial coordinates
The amplitude of f is called intensity or gray level at the point (x, y)

Digital Image Processing
— process digital images by means of a computer, it covers low-, mid-, and high-level processes
low-level: inputs and outputs are images
mid-level: outputs are attributes extracted from input images
high-level: an ensemble of recognition of individual objects

Pixel
— the elements of a digital image

Origins of Digital Image Processing

FIGURE 1.1 A digital picture produced in 1921 from a coded tape by a telegraph printer with special type faces. (McFarlane.†)

Sent by submarine cable between London and New York, the transportation time was reduced to less than three hours from more than a week

Origins of Digital Image Processing

FIGURE 1.4 The first picture of the moon by a U.S. spacecraft. *Ranger 7* took this image on July 31, 1964 at 9 : 09 A.M. EDT, about 17 minutes before impacting the lunar surface. (Courtesy of NASA.)

Sources for Images

- Electromagnetic (EM) energy spectrum
- Acoustic
- Ultrasonic
- Electronic
- Synthetic images produced by computer

Electromagnetic (EM) energy spectrum

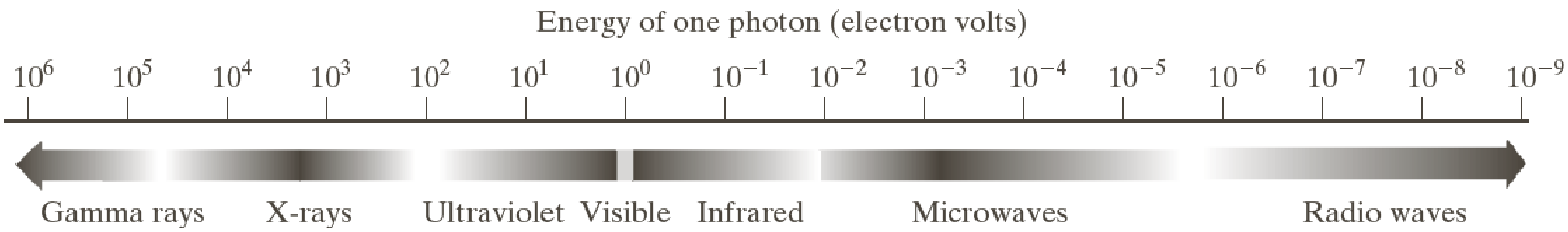


FIGURE 1.5 The electromagnetic spectrum arranged according to energy per photon.

Major uses

Gamma-ray imaging: nuclear medicine and astronomical observations

X-rays: medical diagnostics, industry, and astronomy, etc.

Ultraviolet: lithography, industrial inspection, microscopy, lasers, biological imaging, and astronomical observations

Visible and infrared bands: light microscopy, astronomy, remote sensing, industry, and law enforcement

Microwave band: radar

Radio band: medicine (such as MRI) and astronomy

Examples: Gama-Ray Imaging

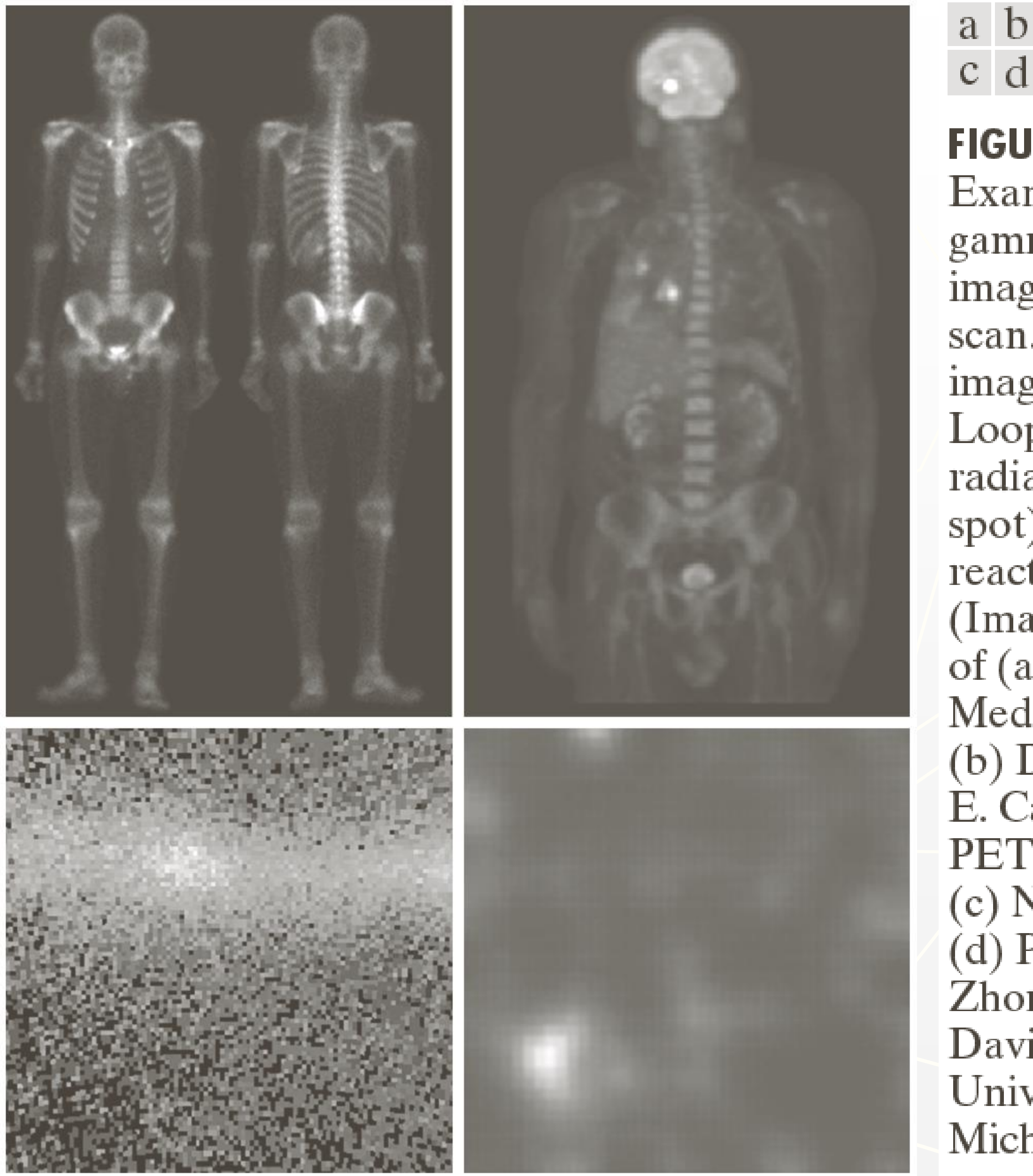


FIGURE 1.6 Examples of gamma-ray imaging. (a) Bone scan. (b) PET image. (c) Cygnus Loop. (d) Gamma radiation (bright spot) from a reactor valve. (Images courtesy of (a) G.E. Medical Systems, (b) Dr. Michael E. Casey, CTI PET Systems, (c) NASA, (d) Professors Zhong He and David K. Wehe, University of Michigan.)

Examples: X-Ray Imaging

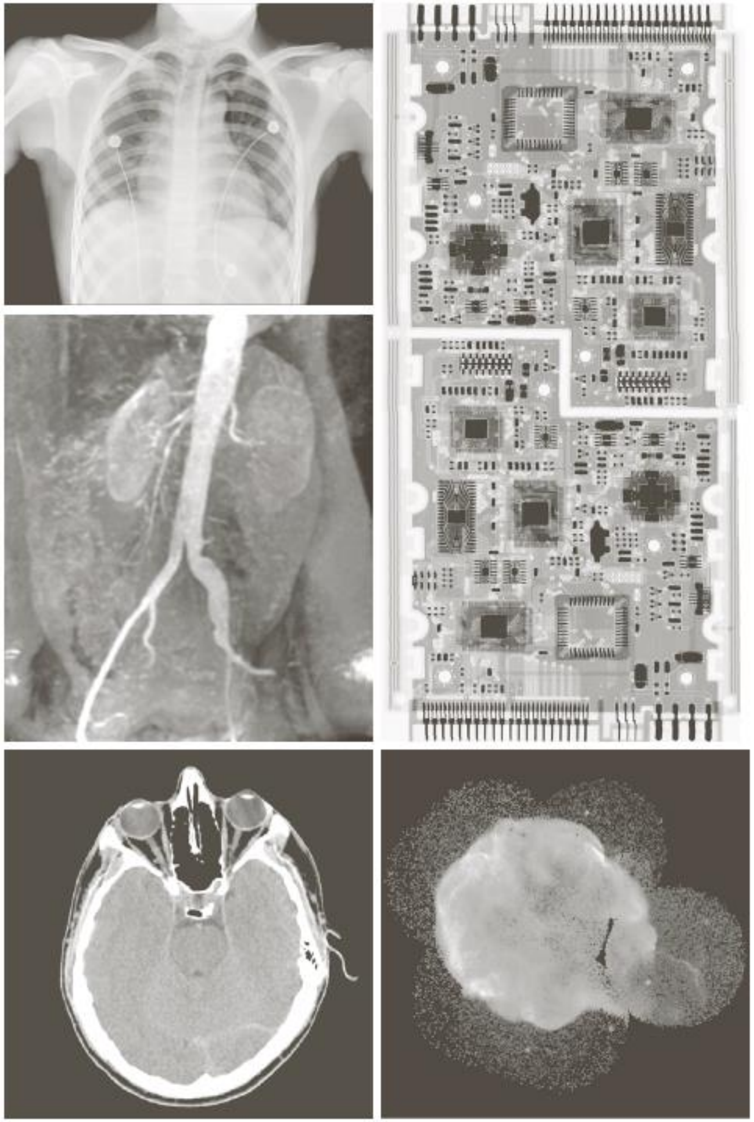


FIGURE 1.7 Examples of X-ray imaging. (a) Chest X-ray. (b) Aortic angiogram. (c) Head CT. (d) Circuit boards. (e) Cygnus Loop. (Images courtesy of (a) and (c) Dr. David R. Pickens, Dept. of Radiology & Radiological Sciences, Vanderbilt University Medical Center; (b) Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School; (d) Mr. Joseph E. Pascente, Lixi, Inc.; and (e) NASA.)

Examples: Visual and Infrared Imaging

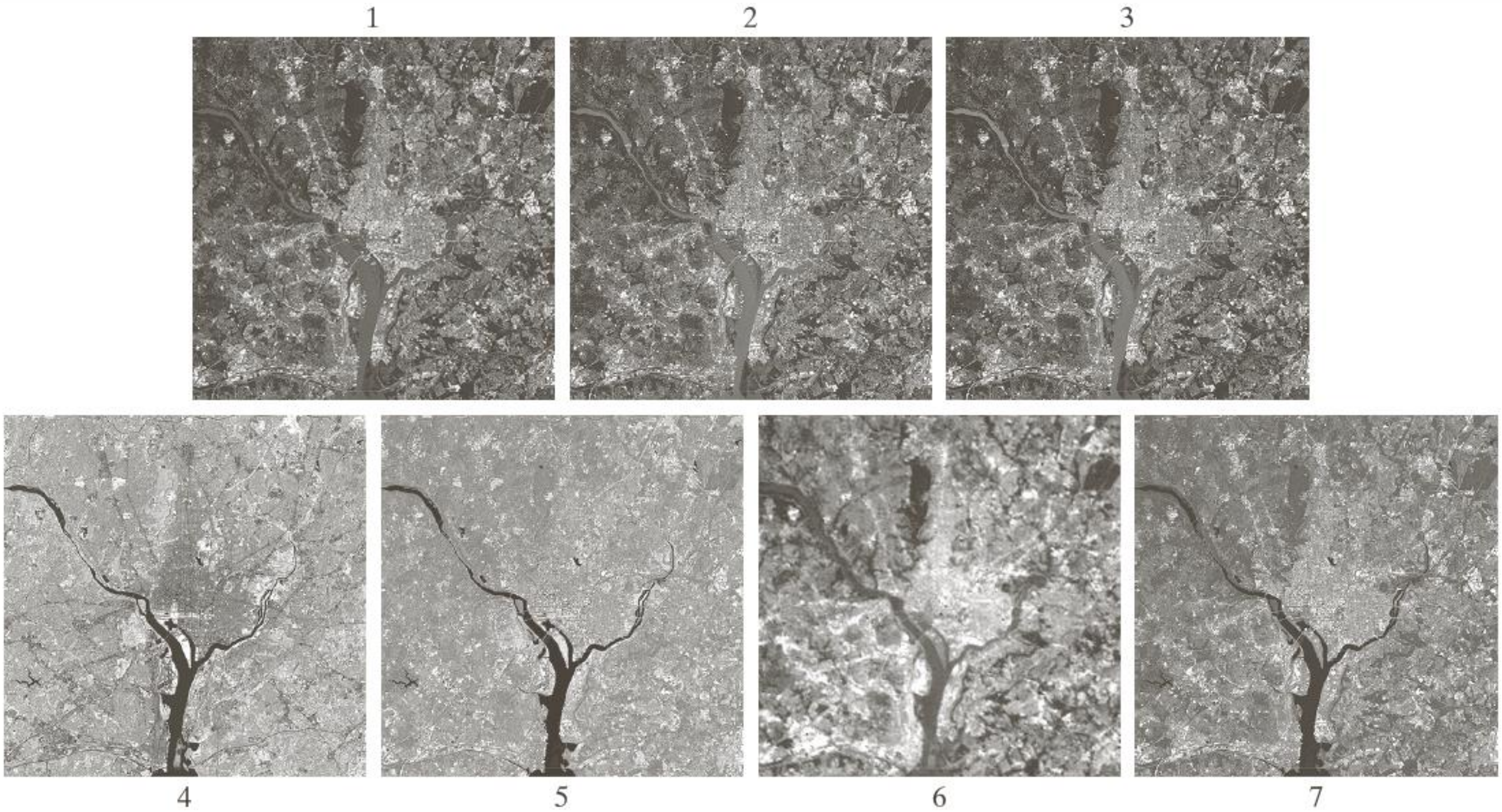


FIGURE 1.10 LANDSAT satellite images of the Washington, D.C. area. The numbers refer to the thematic bands in Table 1.1. (Images courtesy of NASA.)

Examples: Visual and Infrared Imaging

TABLE 1.1
Thematic bands in NASA's LANDSAT satellite.

Band No.	Name	Wavelength (μm)	Characteristics and Uses
1	Visible blue	0.45–0.52	Maximum water penetration
2	Visible green	0.52–0.60	Good for measuring plant vigor
3	Visible red	0.63–0.69	Vegetation discrimination
4	Near infrared	0.76–0.90	Biomass and shoreline mapping
5	Middle infrared	1.55–1.75	Moisture content of soil and vegetation
6	Thermal infrared	10.4–12.5	Soil moisture; thermal mapping
7	Middle infrared	2.08–2.35	Mineral mapping

Examples: Infrared Satellite Imaging

Example: NASA's James Webb Space Telescope

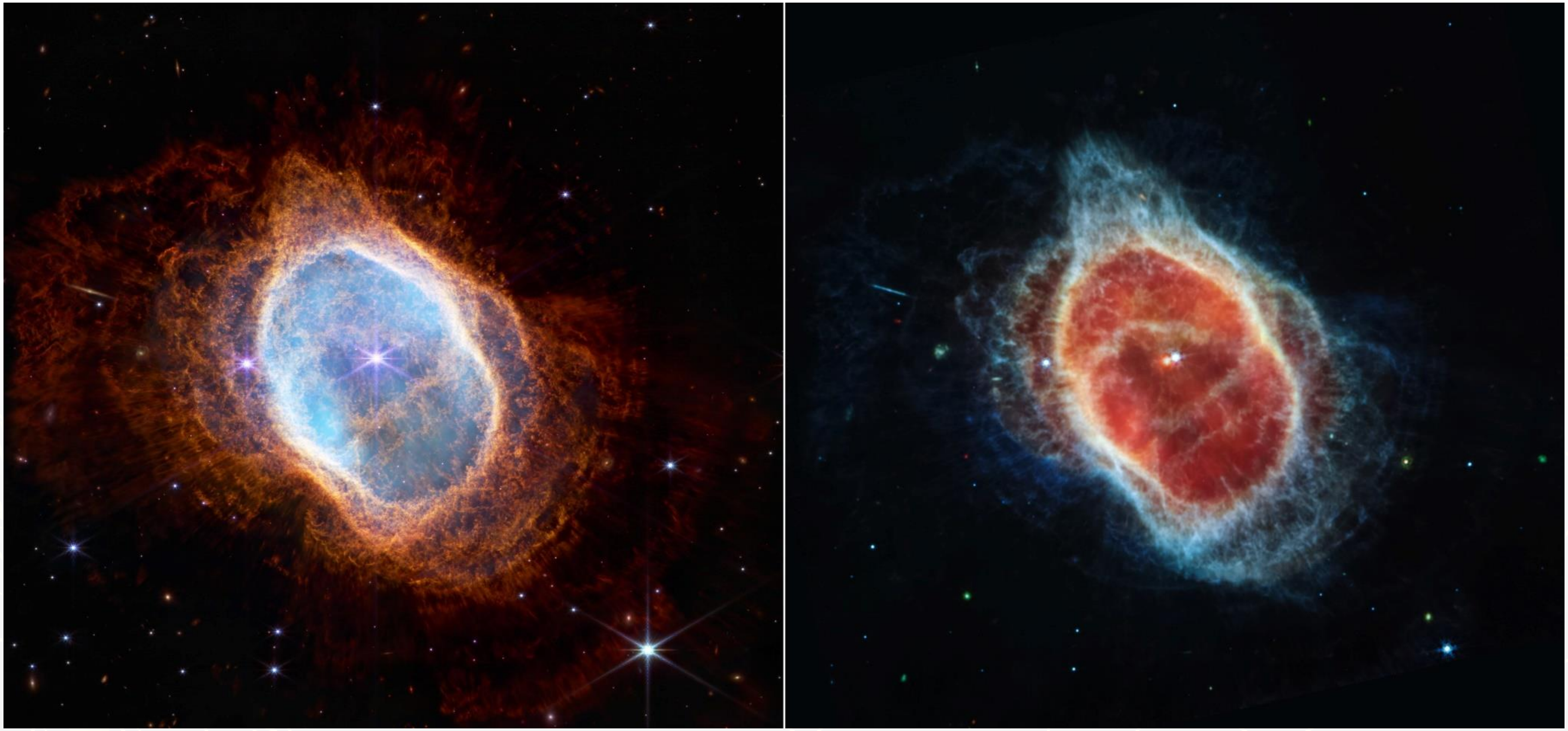


Image of Southern ring planetary nebula. The left side is an image from the NIRcam instrument, the right side is the image from the MIRI instruments

Examples: Automated Visual Inspection

a b
c
d

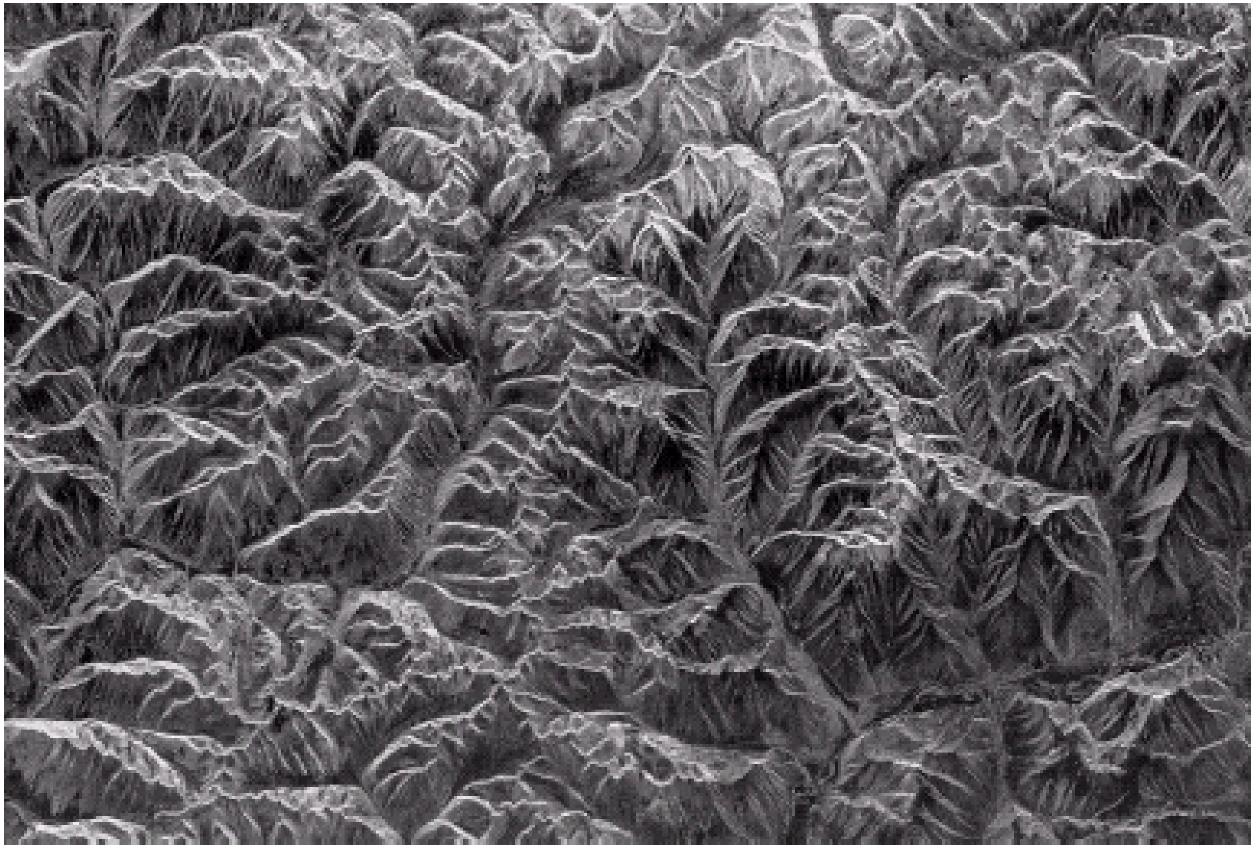
Results of automated reading of the plate content by the system

The area in which the imaging system detected the plate

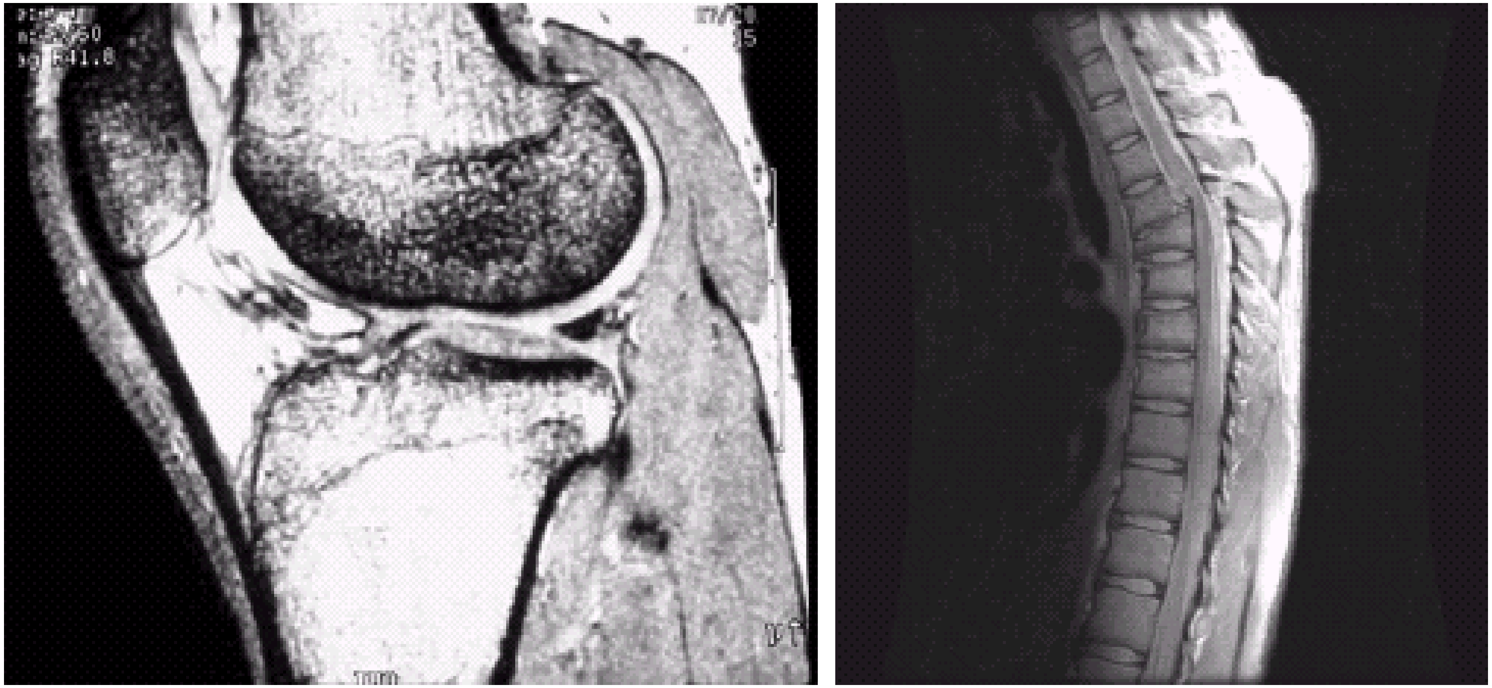
FIGURE 1.15 Some additional examples of imaging in the visual spectrum. (a) Thumb print. (b) Paper currency. (c) and (d) Automated license plate reading. (Figure (a) courtesy of the National Institute of Standards and Technology. Figures (c) and (d) courtesy of Dr. Juan Herrera, Perceptics Corporation.)

Example of Radar Image

FIGURE 1.16 Spaceborne radar image of mountains in southeast Tibet. (Courtesy of NASA.)



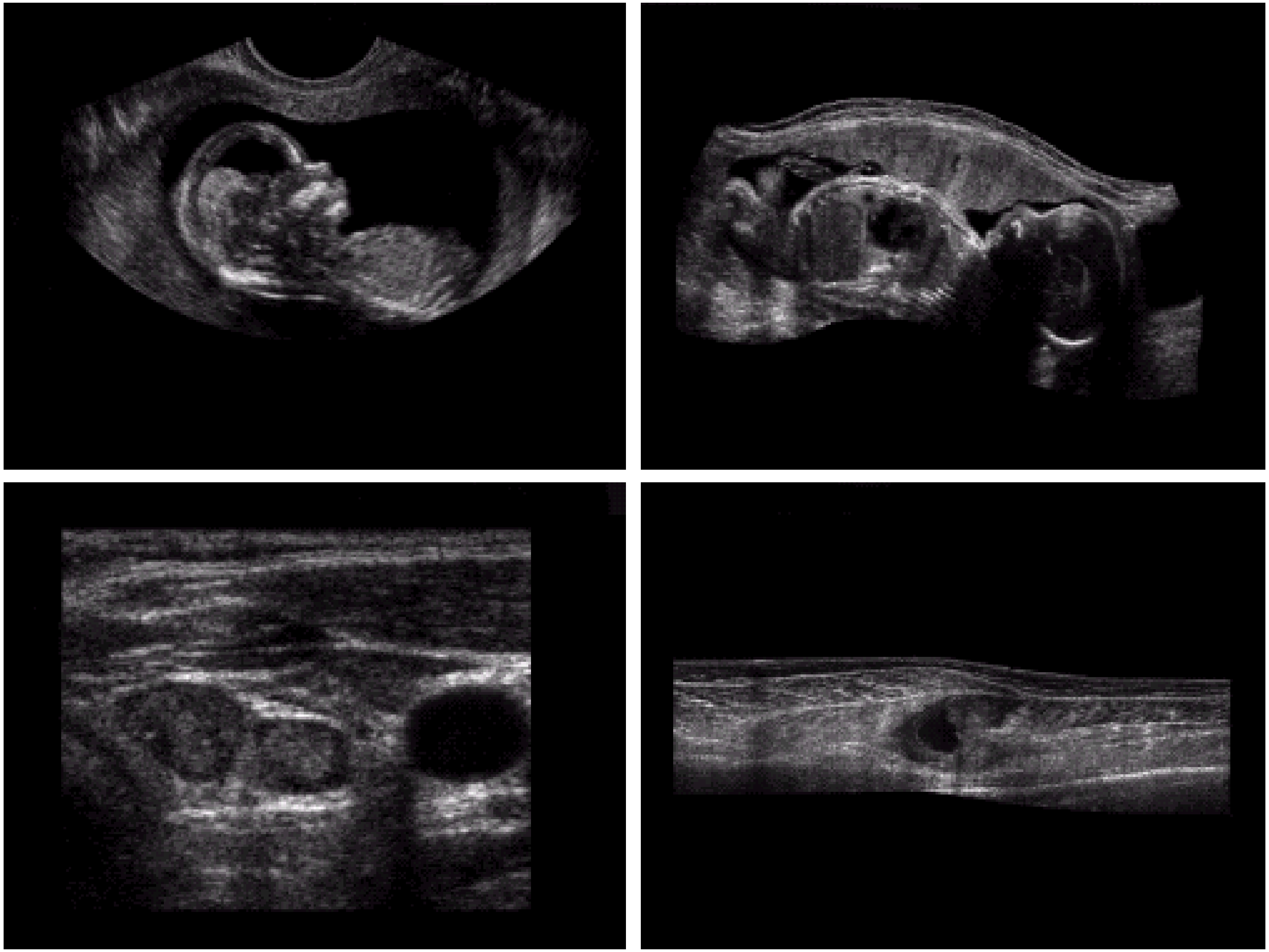
Examples: MRI (Radio Band)



a b

FIGURE 1.17 MRI images of a human (a) knee, and (b) spine. (Image (a) courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School, and (b) Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

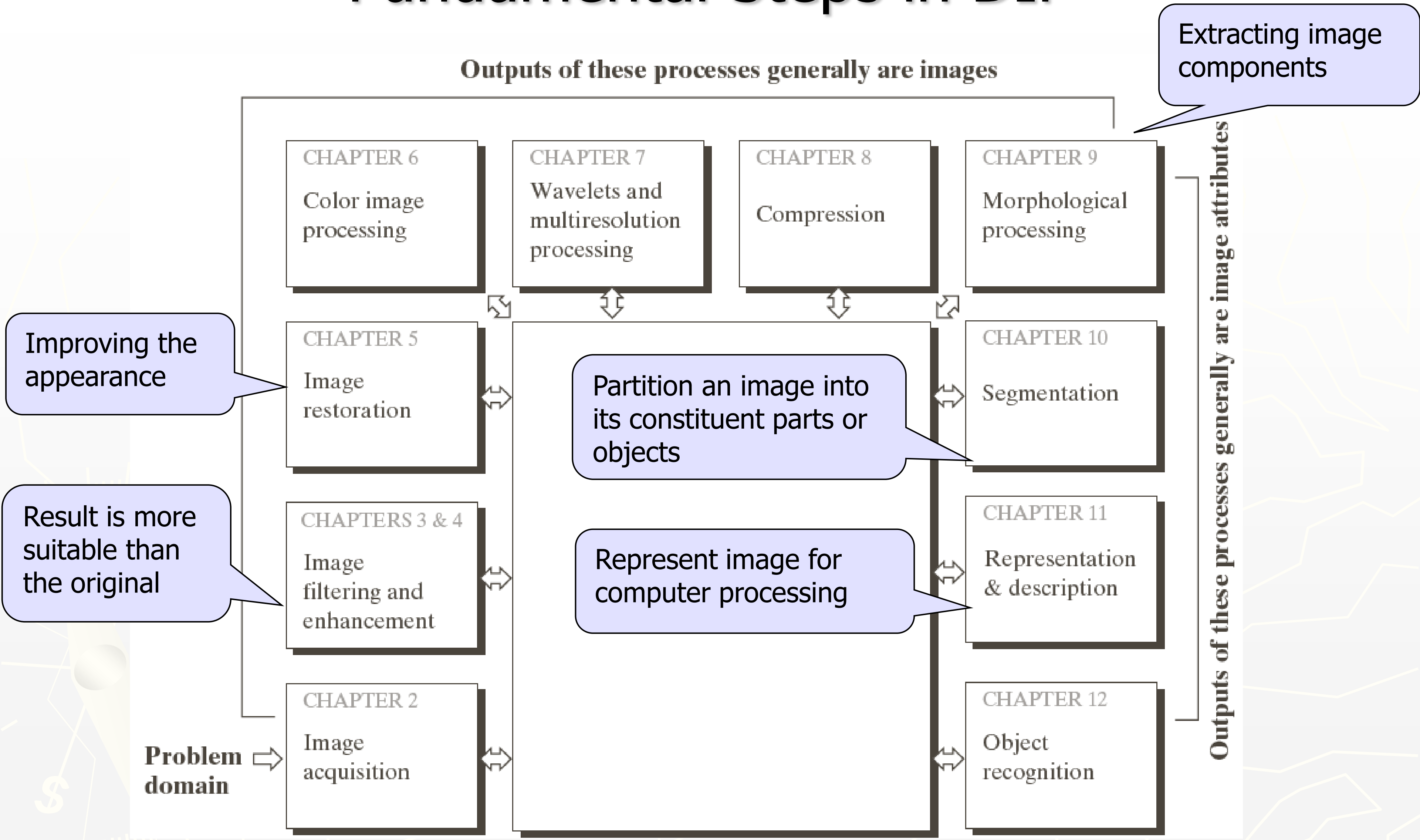
Examples: Ultrasound Imaging



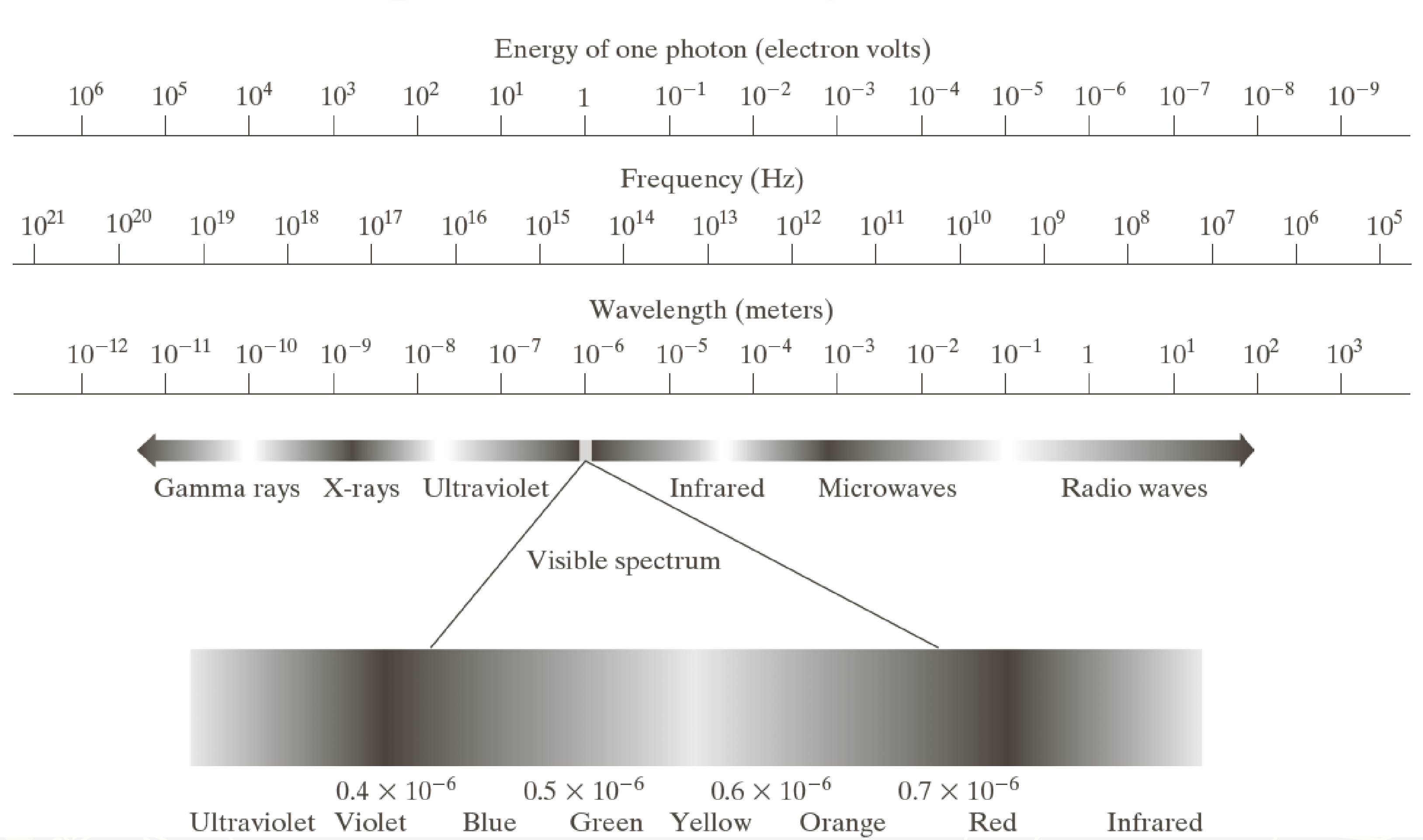
a	b
c	d

FIGURE 1.20 Examples of ultrasound imaging. (a) Baby. (2) Another view of baby. (c) Thyroids. (d) Muscle layers showing lesion. (Courtesy of Siemens Medical Systems, Inc., Ultrasound Group.)

Fundamental Steps in DIP



Light and EM Spectrum



Light and EM Spectrum

- The colors that humans perceive in an object are determined by the nature of the light reflected from the object.

  e.g. green objects reflect light with wavelengths primarily in the 500 to 570 nm range while absorbing most of the energy at other wavelength

Light and EM Spectrum

- Monochromatic light: void of color

  Intensity is the only attribute, from black to white

  Monochromatic images are referred to as **gray-scale** images

- Chromatic light bands: 0.43 to 0.79 um

  The quality of a chromatic light source:

  Radiance: total amount of energy

  Luminance (lm): the amount of energy an observer perceives from a light source

  Brightness: a subjective descriptor of light perception that is impossible to measure. It embodies the achromatic notion of intensity and one of the key factors in describing color sensation.

Image Acquisition

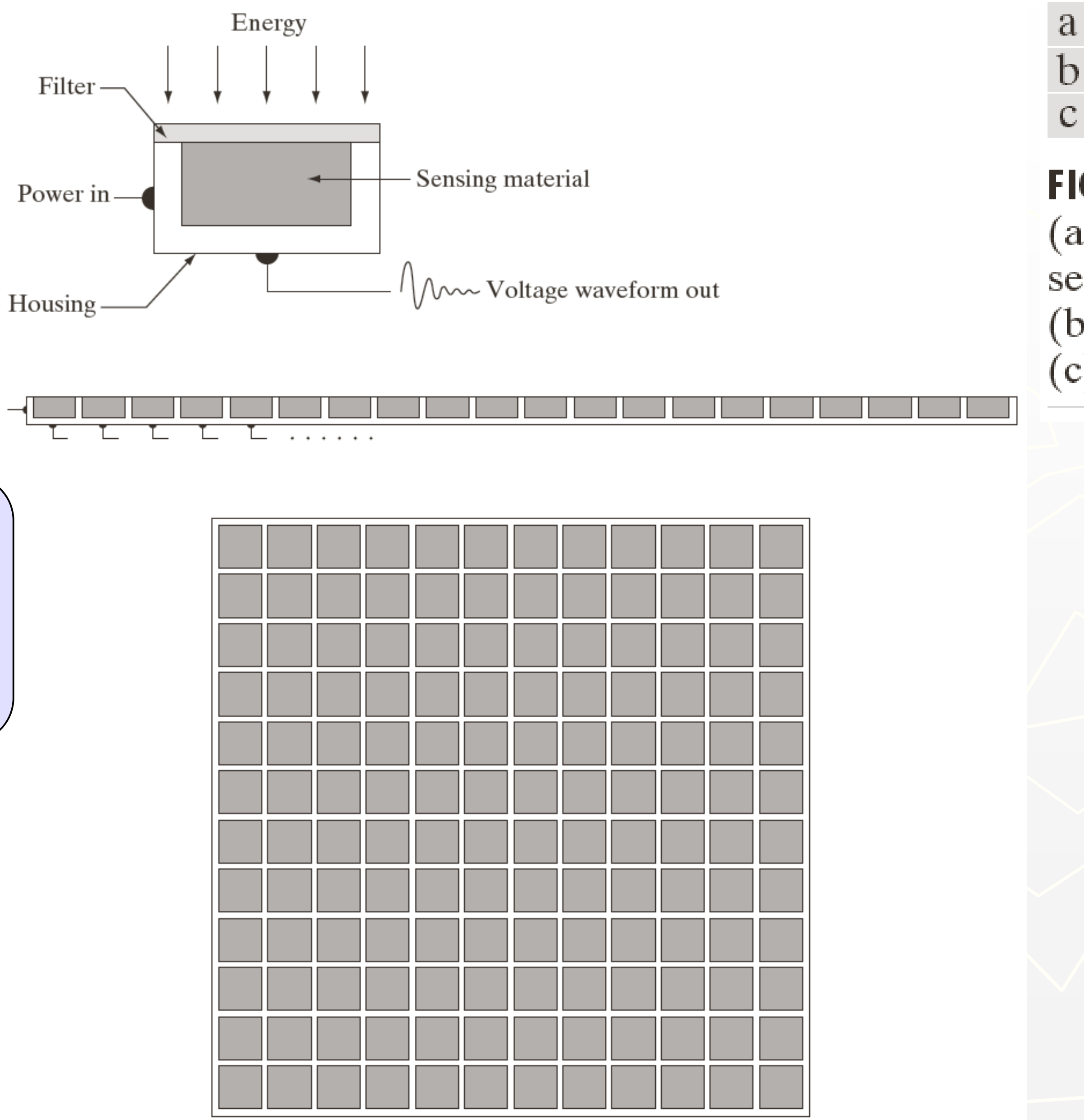


FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Transform illumination energy into digital images

Image Acquisition Process

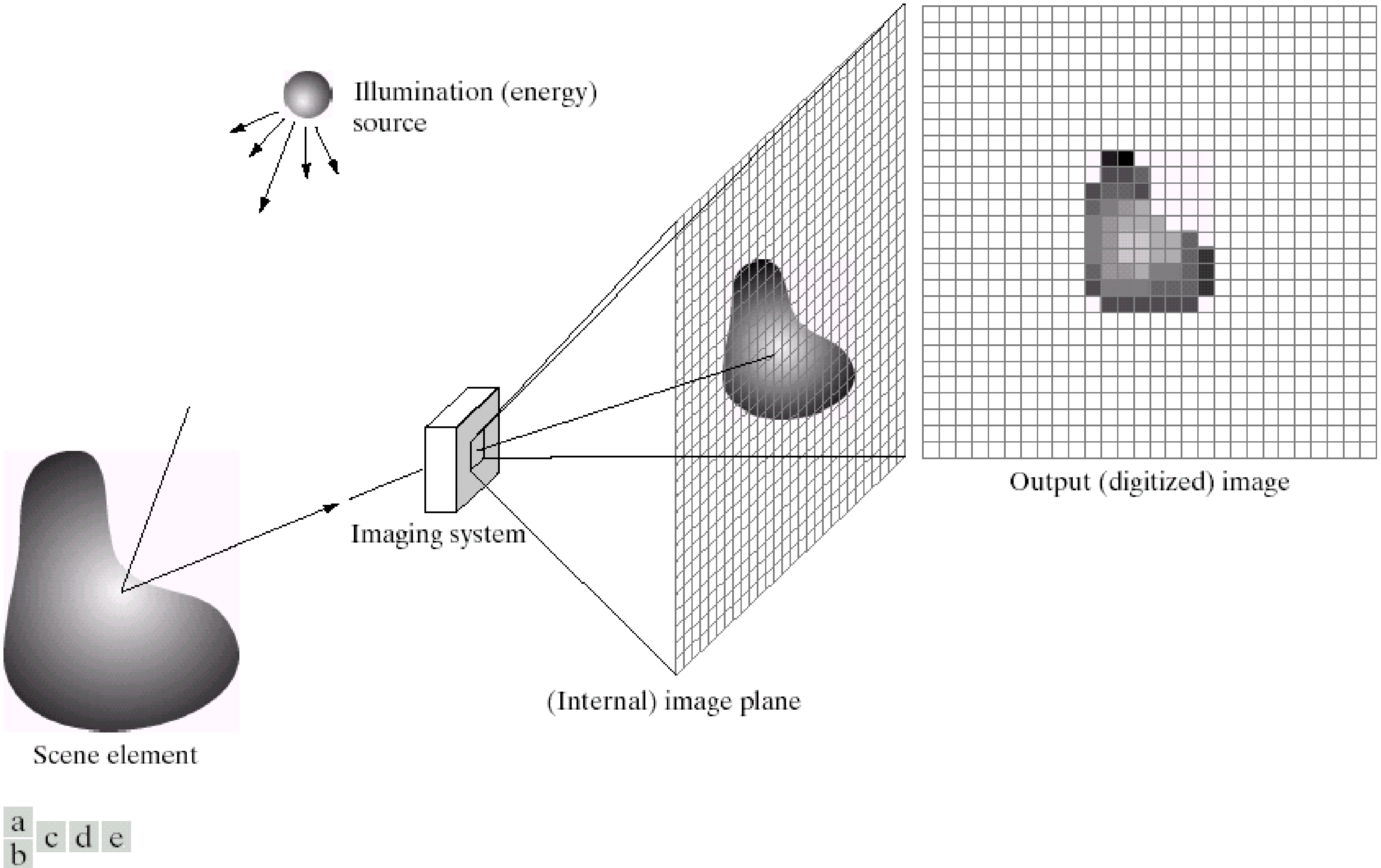


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

A Simple Image Formation Model

$$f(x, y) = i(x, y) \cdot r(x, y)$$

$f(x, y)$: intensity at the point (x, y)

$i(x, y)$: illumination at the point (x, y)

(the amount of source illumination incident on the scene)

$r(x, y)$: reflectance/transmissivity at the point (x, y)

(the amount of illumination reflected/transmitted by the object)

where $0 < i(x, y) < \infty$ and $0 < r(x, y) < 1$

Some Typical Ranges of illumination

- Illumination

  Lumen — A unit of light flow or luminous flux

  Lumen per square meter (lm/m^2) — The metric unit of measure for illuminance of a surface

  - On a clear day, the sun may produce in excess of 90,000 lm/m^2 of illumination on the surface of the Earth
  - On a cloudy day, the sun may produce less than 10,000 lm/m^2 of illumination on the surface of the Earth
  - On a clear evening, the moon yields about 0.1 lm/m^2 of illumination
  - The typical illumination level in a commercial office is about 1000 lm/m^2

Some Typical Ranges of Reflectance

- Reflectance
  - 0.01 for black velvet
  - 0.65 for stainless steel
  - 0.80 for flat-white wall paint
  - 0.90 for silver-plated metal
  - 0.93 for snow

Image Sampling and Quantization

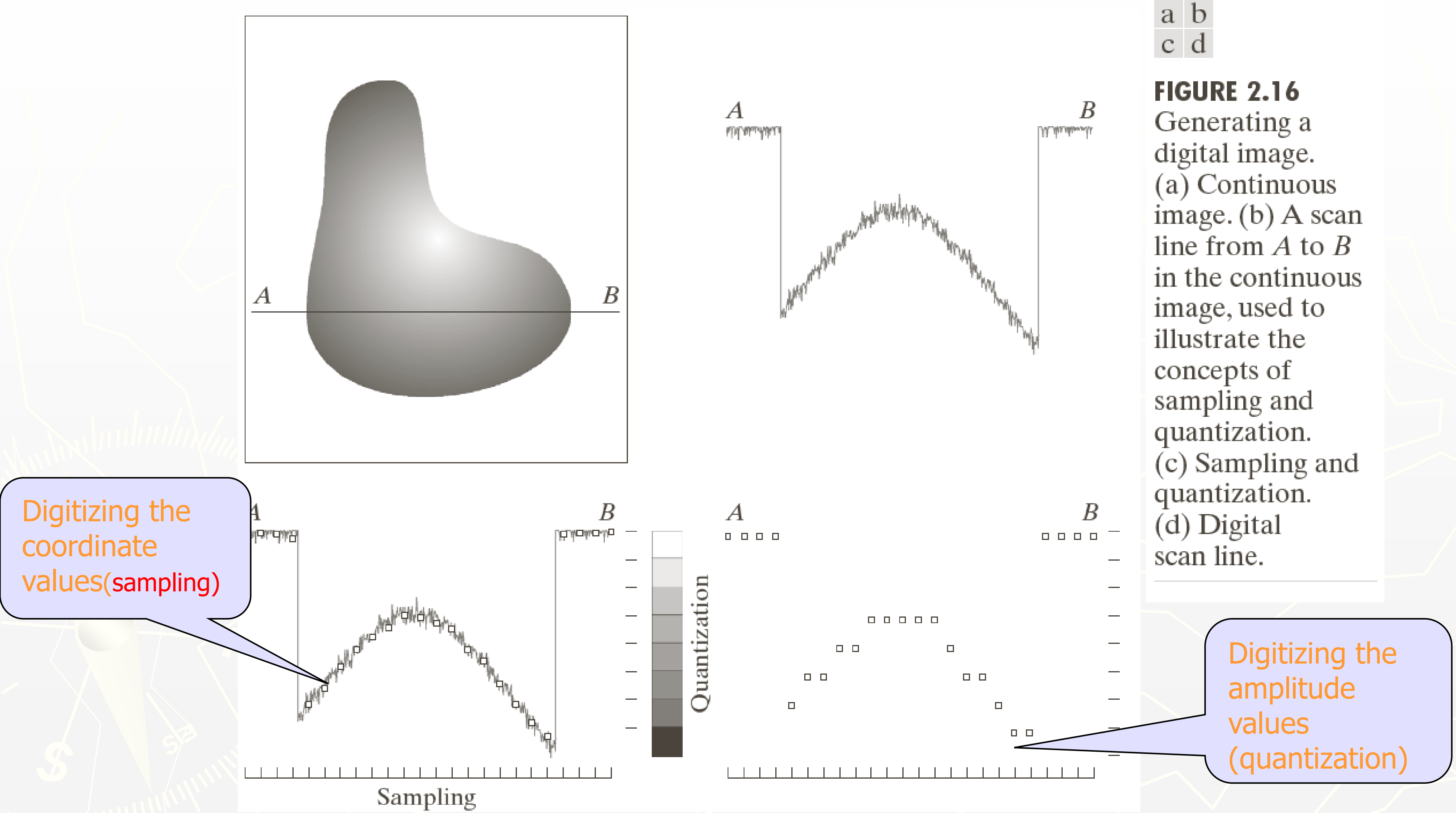


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Image Sampling and Quantization

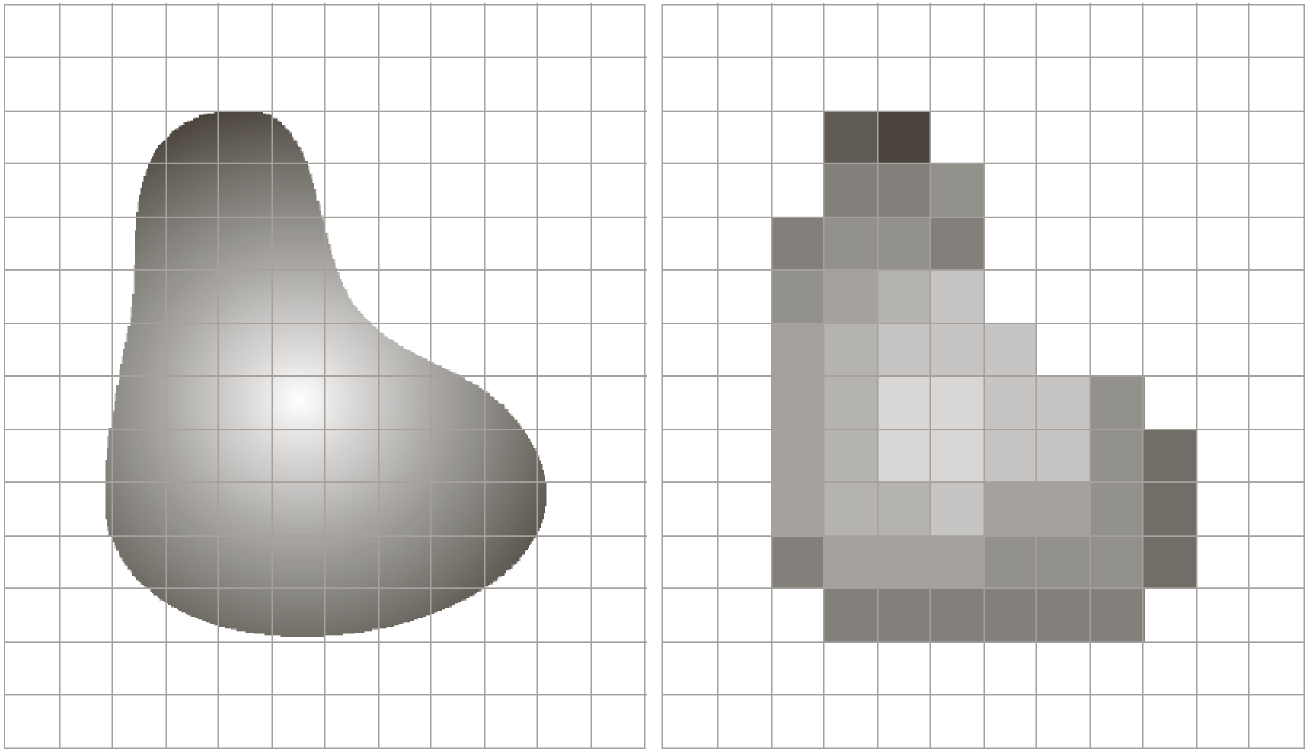


FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Representing Digital Images

- The representation of an M×N numerical array as

$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & \ldots & f(0,N-1) \\ f(1,0) & f(1,1) & \ldots & f(1,N-1) \\ \ldots & \ldots & \ldots & \ldots \\ f(M-1,0) & f(M-1,1) & \ldots & f(M-1,N-1) \end{bmatrix}$$

Representing Digital Images

- The representation of an M×N numerical array as

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Representing Digital Images

- The representation of an M×N numerical array in MATLAB

$$f(x,y)=\begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

Representing Digital Images

- Discrete intensity interval $[0, L-1]$, $L=2^k$
- The number b of bits required to store a $M \times N$ digitized image

$$b = M \times N \times k$$

Image Interpolation

- **Interpolation** — Process of using known data to estimate unknown values

  e.g., zooming, shrinking, rotating, and geometric correction

- **Interpolation** (sometimes called ***resampling***) — an imaging method to increase (or decrease) the number of pixels in a digital image.

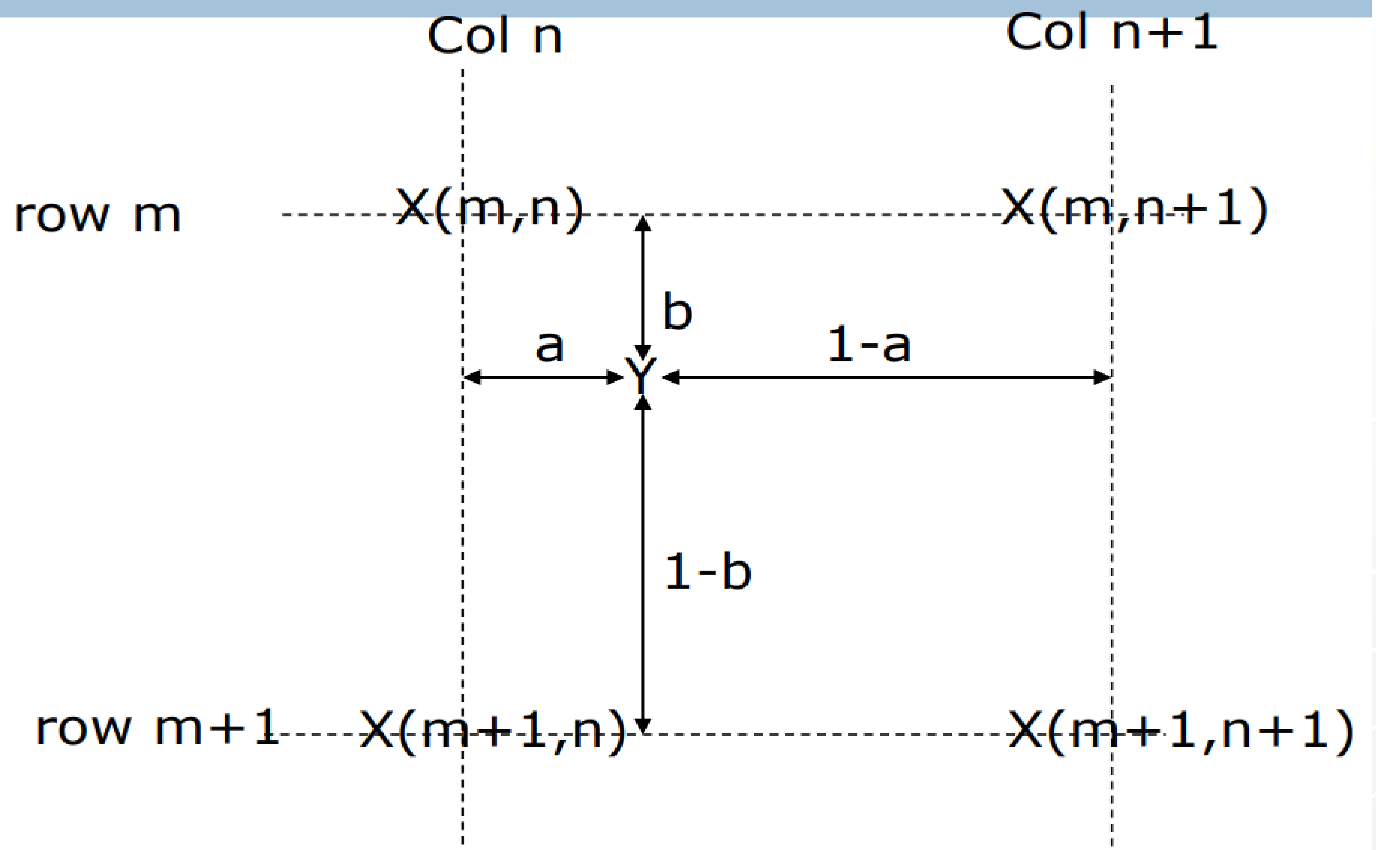
  Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

Image Interpolation: Nearest Neighbor Interpolation

Nearest neighbor is the most basic and requires the least processing time of all the interpolation algorithms because it only considers one pixel — the closest one to the interpolated point. This has the effect of simply making each pixel bigger.

Image Interpolation:

Bilinear Interpolation



Q: what is the interpolated value at Y?

Ans.: $(1-a)(1-b)X(m,n)+(1-a)bX(m+1,n)$
$+a(1-b)X(m,n+1)+abX(m+1,n+1)$

Image Interpolation: Bicubic Interpolation

- The intensity value assigned to point (x,y) is obtained by the following equation

$$f_3(x,y)=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}x^iy^j$$

- The sixteen coefficients are determined by using the sixteen nearest neighbors.

http://en.wikipedia.org/wiki/Bicubic_interpolation

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Examples: Interpolation

Examples: Interpolation

Nearest Neighbor Interpolation

Examples: Interpolation

Examples: Interpolation

Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

Basic Relationships Between Pixels

- **Neighbors** of a pixel p at coordinates (x,y)

  - **4-neighbors of p**, denoted by $\mathbf{N_4(p)}$:
    (x-1, y), (x+1, y), (x,y-1), and (x, y+1).

  - **4 diagonal neighbors of p**, denoted by $\mathbf{N_D(p)}$:
    (x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).

  - **8 neighbors of p**, denoted $\mathbf{N_8(p)}$
    $N_8(p) = N_4(p) \cup N_D(p)$

Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values used to define adjacency

- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

- **8-adjacency**: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values

- **m-adjacency**: Two pixels p and q with values from V are m-adjacent if

(i) q is in the set $N_4(p)$, or

(ii) q is in the set $\mathbf{N_D(p)}$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

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Basic Relationships Between Pixels

- **Path**
- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$.

- Here *n* is the *length* of the path.
- If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.
- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent **m-adjacent**

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{matrix}$$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

m-adjacent

The 8-path from (1,3) to (3,3):
(i) (1,3), (1,2), (2,2), (3,3)
(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):
(1,3), (1,2), (2,2), (3,3)

Basic Relationships Between Pixels

- **Connected in S**

  Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

  $$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

  Where $\forall i, 0 \le i \le n, (x_i, y_i) \in S$

Basic Relationships Between Pixels

- **Boundary (or border)**

- The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
- If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

Question 1

- **In the following arrangement of pixels, are the two regions (of 1s) adjacent? (if 8-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

Question 2

- **In the following arrangement of pixels, are the two parts (of 1s) adjacent? (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Part 1

Part 2

- **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

Question 3

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 8-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Question 4

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 4-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Distance Measures

- Given pixels *p, q* and *z* with coordinates (x, y), (s, t), (u, v) respectively, the distance function D has following properties:

a. $D(p, q) \geq 0 \quad [D(p, q) = 0, \text{ iff } p = q]$

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$

Distance Measures

The following are the different Distance measures:

a. Euclidean Distance :

$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$

b. City Block Distance:

$D_4(p, q) = |x-s| + |y-t|$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

c. Chess Board Distance:

$D_8(p, q) = \max(|x-s|, |y-t|)$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Question 5

- **In the following arrangement of pixels, what's the value of the chessboard distance between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 6

- **In the following arrangement of pixels, what's the value of the city-block distance between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Introduction to Mathematical Operations in DIP

- **Array vs. Matrix Operation**

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product operator

$$A \,.\!*\, B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix product operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Introduction to Mathematical Operations in DIP

- **Linear vs. Nonlinear Operation**

$$H\left[f(x,y)\right]=g(x,y)$$

$$H\left[a_i f_i(x,y)+a_j f_j(x,y)\right]$$

$$=H\left[a_i f_i(x,y)\right]+H\left[a_j f_j(x,y)\right]$$

Additivity

$$=a_i H\left[f_i(x,y)\right]+a_j H\left[f_j(x,y)\right]$$

Homogeneity

$$=a_i g_i(x,y)+a_j g_j(x,y)$$

H is said to be a **linear operator;**

H is said to be a **nonlinear operator** if it does not meet the above qualification.

Arithmetic Operations

- Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

$$s(x,y) = f(x,y) + g(x,y)$$
$$d(x,y) = f(x,y) - g(x,y)$$
$$p(x,y) = f(x,y) \times g(x,y)$$
$$v(x,y) = f(x,y) \div g(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

Noiseless image: f(x,y)

Noise: n(x,y) (at every pair of coordinates (x,y), the noise is uncorrelated and has zero average value)

Corrupted image: g(x,y)

$$g(x,y) = f(x,y) + n(x,y)$$

Reducing the noise by adding a set of noisy images, $\{g_i(x,y)\}$

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

$$E\{\bar{g}(x,y)\} = E\left\{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)\right\}$$

$$= E\left\{\frac{1}{K}\sum_{i=1}^{K}\left[f(x,y)+n_i(x,y)\right]\right\}$$

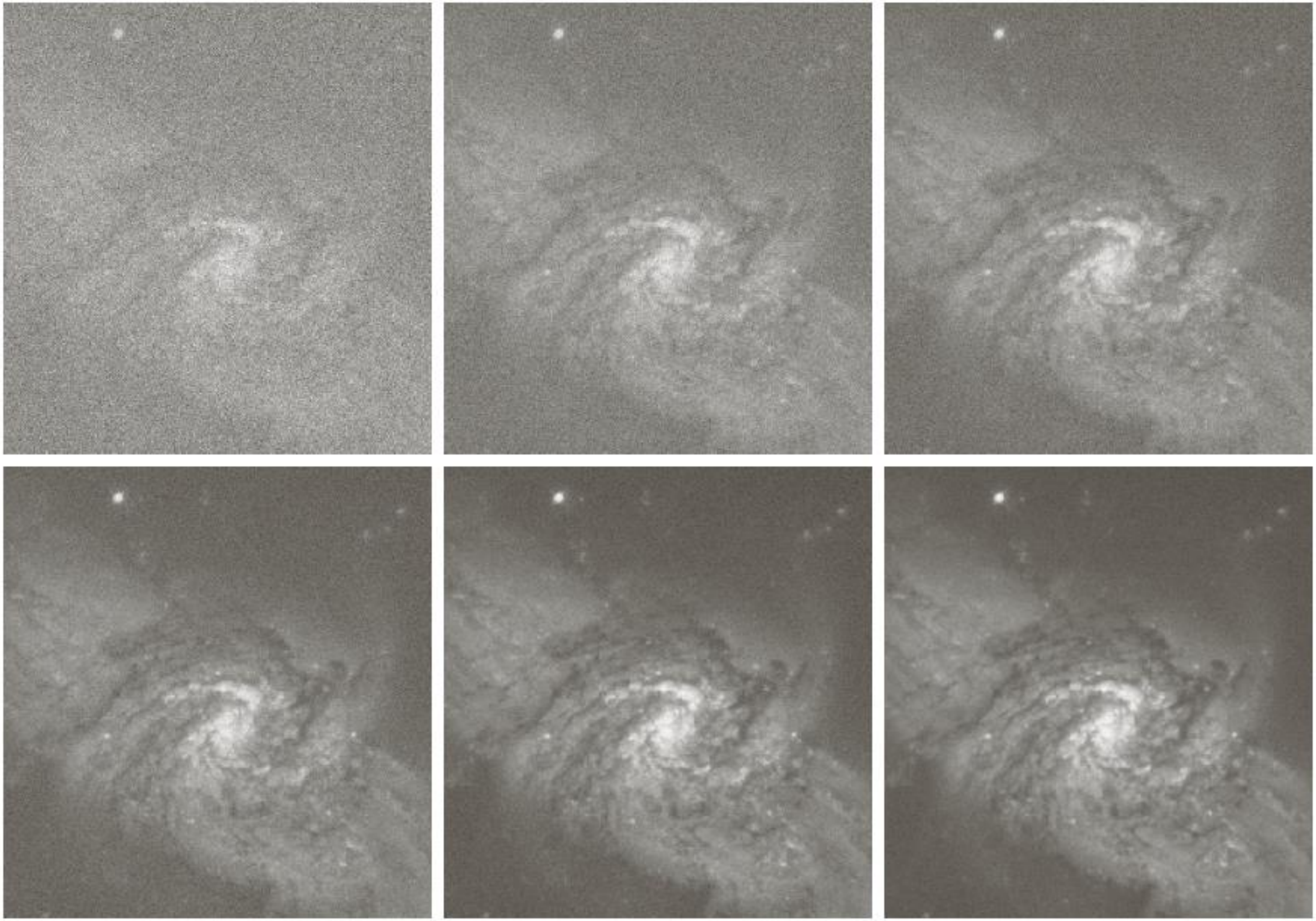
$$= f(x,y) + E\left\{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)\right\}$$

$$= f(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

- In astronomy, imaging under very low light levels frequently causes sensor noise to render single images virtually useless for analysis.

- In astronomical observations, similar sensors for noise reduction by observing the same scene over long periods of time. Image averaging is then used to reduce the noise.



a b c
d e f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

An Example of Image Subtraction: Mask Mode Radiography

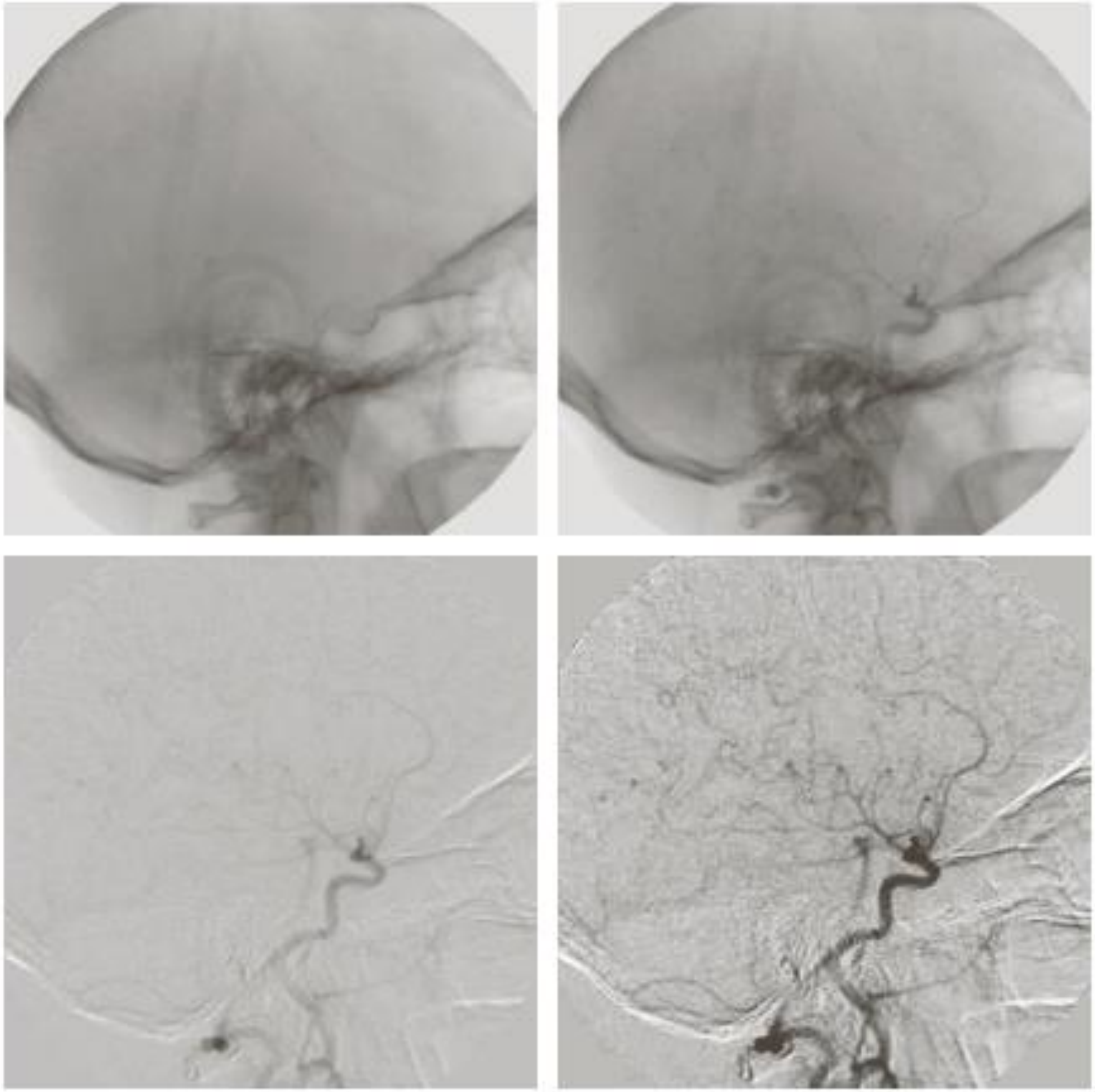
Mask $h(x,y)$: an X-ray image of a region of a patient's body

Live images $f(x,y)$: X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail $g(x,y)$

$$g(x,y) = f(x,y) - h(x,y)$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.



a	b
c	d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

An Example of Image Multiplication

a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Set and Logical Operations

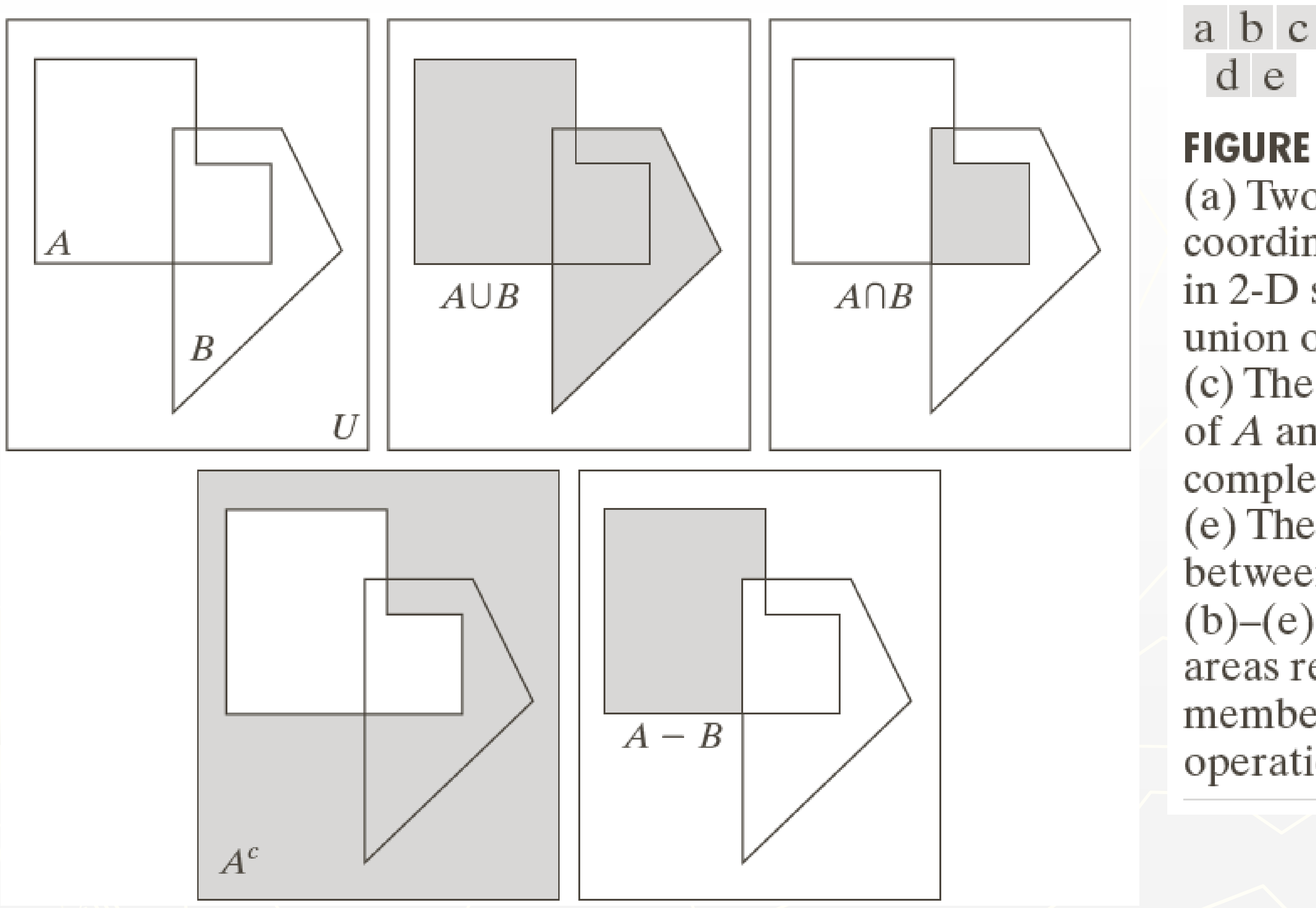


FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Set and Logical Operations

- Let A be the elements of a gray-scale image

  The elements of A are triplets of the form (x, y, z), where x and y are spatial coordinates and z denotes the intensity at the point (x, y).

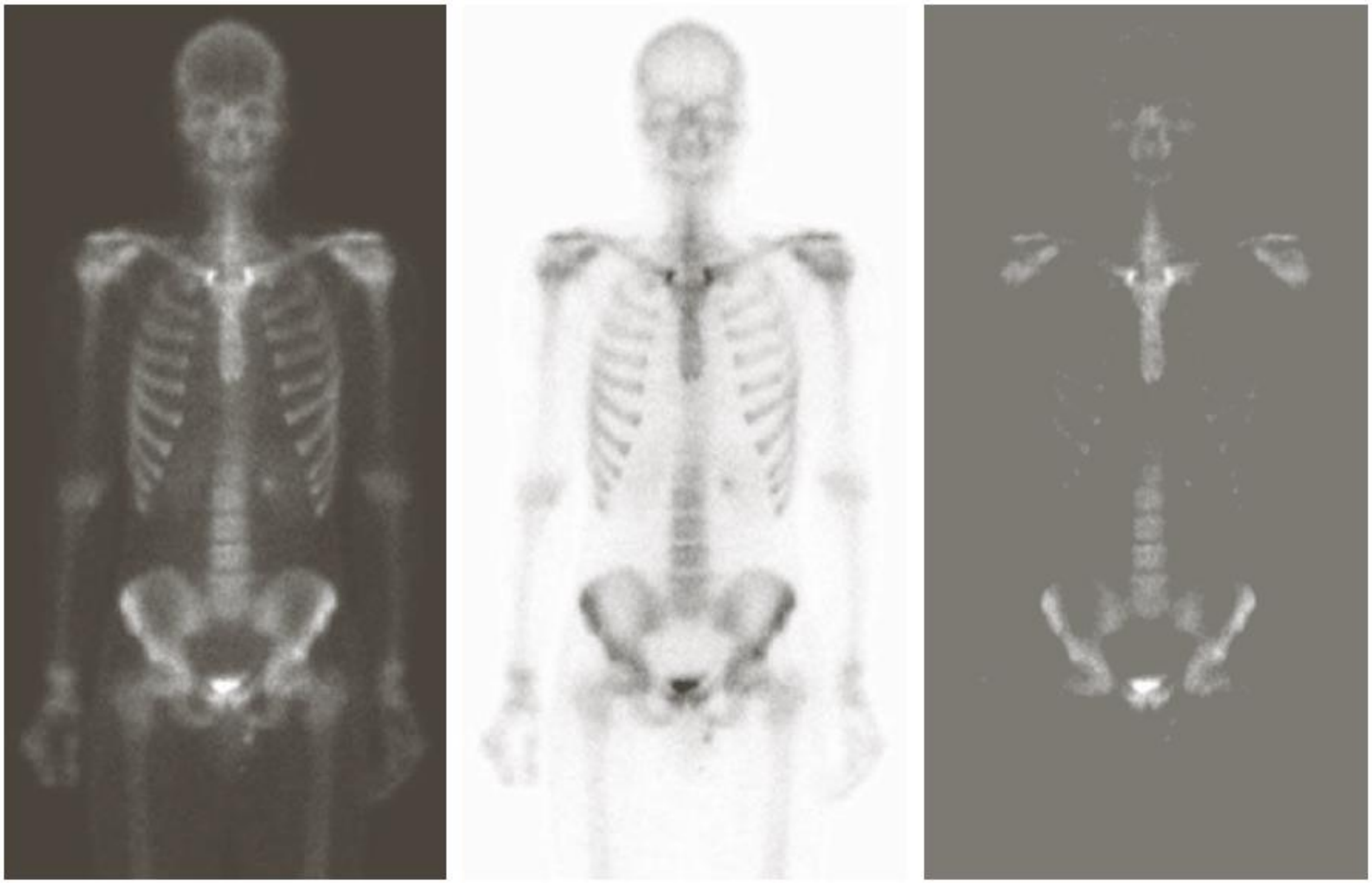
$$A = \{(x, y, z) \mid z = f(x, y)\}$$

- The complement of A is denoted A^c

$$A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

$$K = 2^k - 1; k \text{ is the number of intensity bits used to represent } z$$

Set and Logical Operations



a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Set and Logical Operations

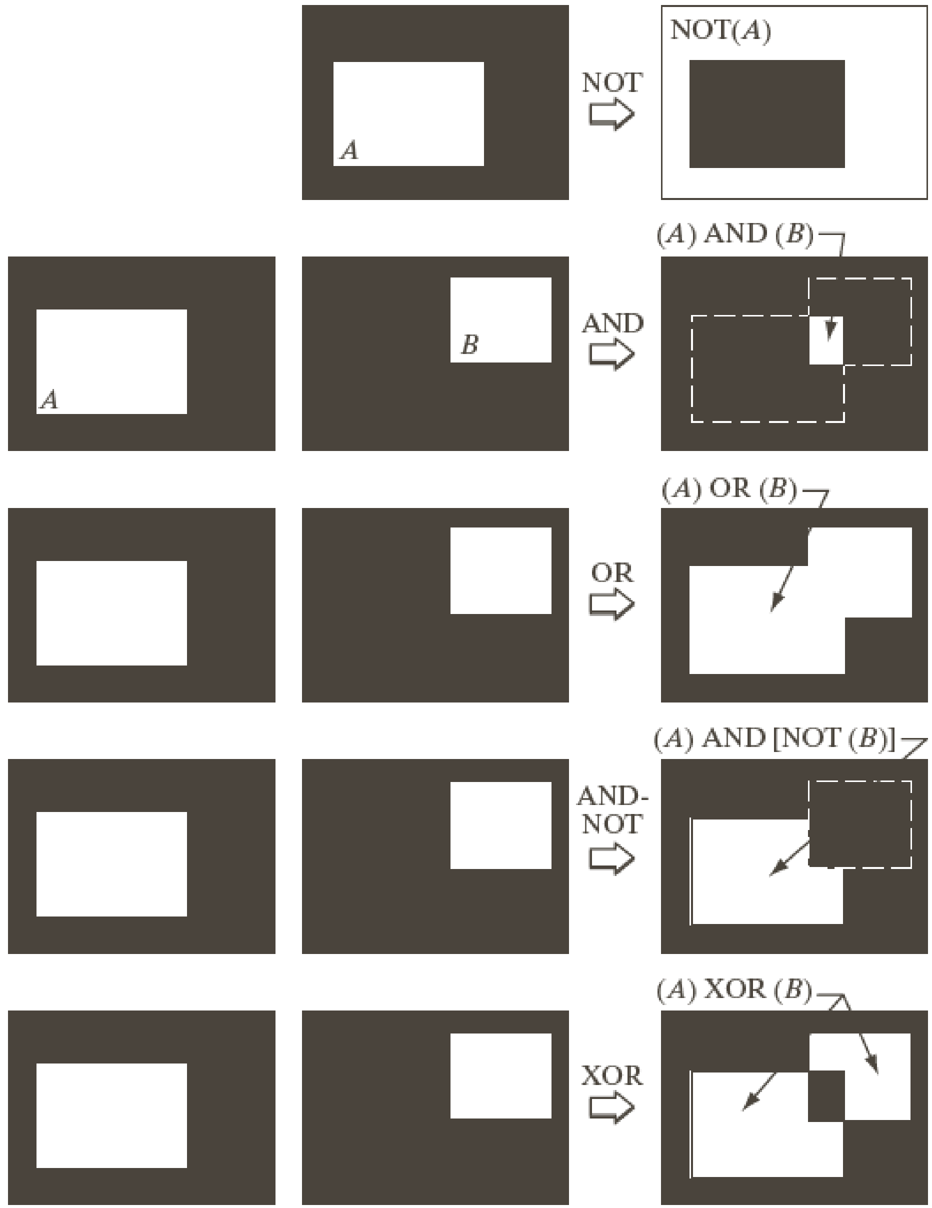


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

Spatial Operations

- Single-pixel operations

Alter the values of an image's pixels based on the intensity.

$$s = T(z)$$

e.g.,

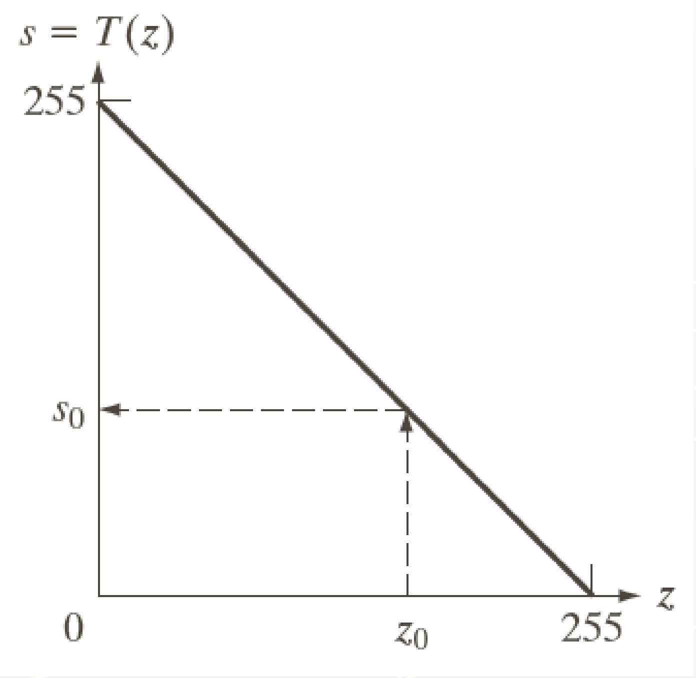


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Geometric Spatial Transformations

- Geometric transformation (rubber-sheet transformation)

  — A spatial transformation of coordinates

$$(x, y) = T\{(v, w)\}$$

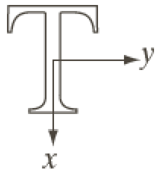
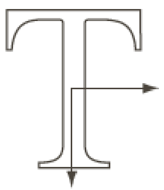
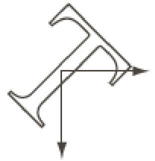
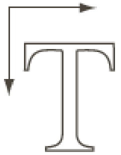
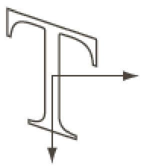
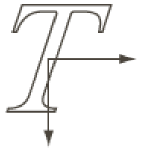
  — intensity interpolation that assigns intensity values to the spatially transformed pixels.

- Affine transform

$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

TABLE 2.2

Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Intensity Assignment

- Forward Mapping

$$(x, y) = T\{(v, w)\}$$

It's possible that two or more pixels can be transformed to the same location in the output image.

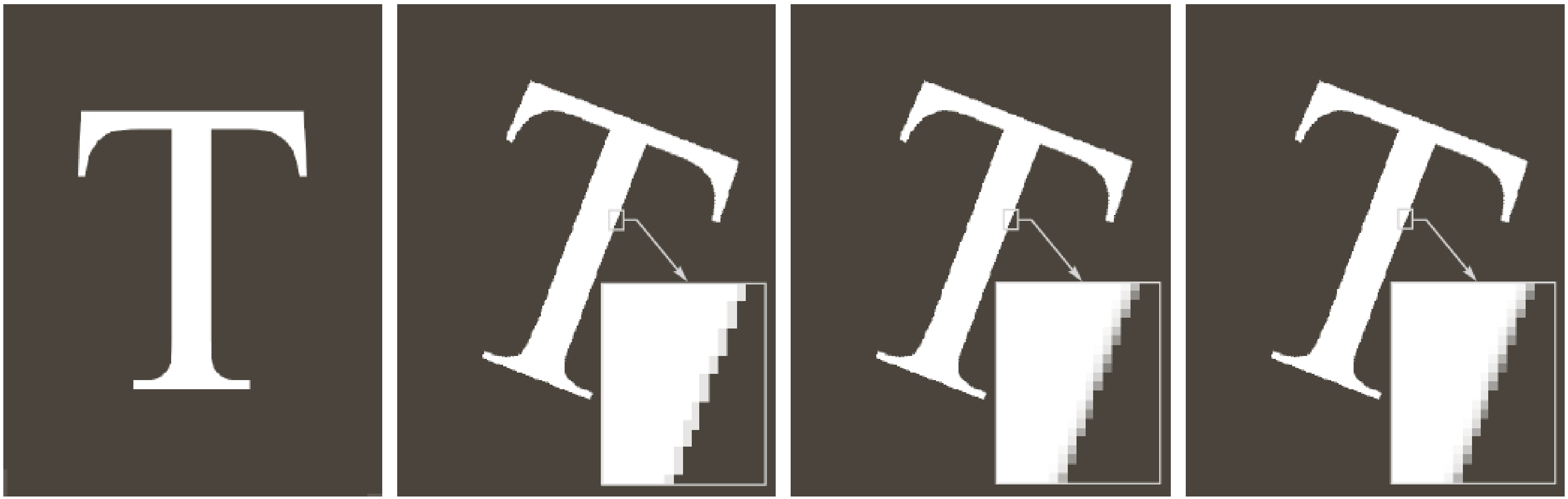
- Inverse Mapping

$$(v, w) = T^{-1}\{(x, y)\}$$

The nearest input pixels to determine the intensity of the output pixel value.

Inverse mappings are more efficient to implement than forward mappings.

Example: Image Rotation and Intensity Interpolation



a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Image Registration

- Input and output images are available but the transformation function is unknown.

  Goal: estimate the transformation function and use it to register the two images.

- One of the principal approaches for image registration is to use ***tie points*** (also called ***control points***)
  - The corresponding points are known precisely in the input and output (**reference**) images.

Image Registration

- A simple model based on bilinear approximation:

$$\begin{cases} x = c_1 v + c_2 w + c_3 vw + c_4 \\ y = c_5 v + c_6 w + c_7 vw + c_8 \end{cases}$$

Where (v, w) and (x, y) are the coordinates of tie points in the input and reference images.

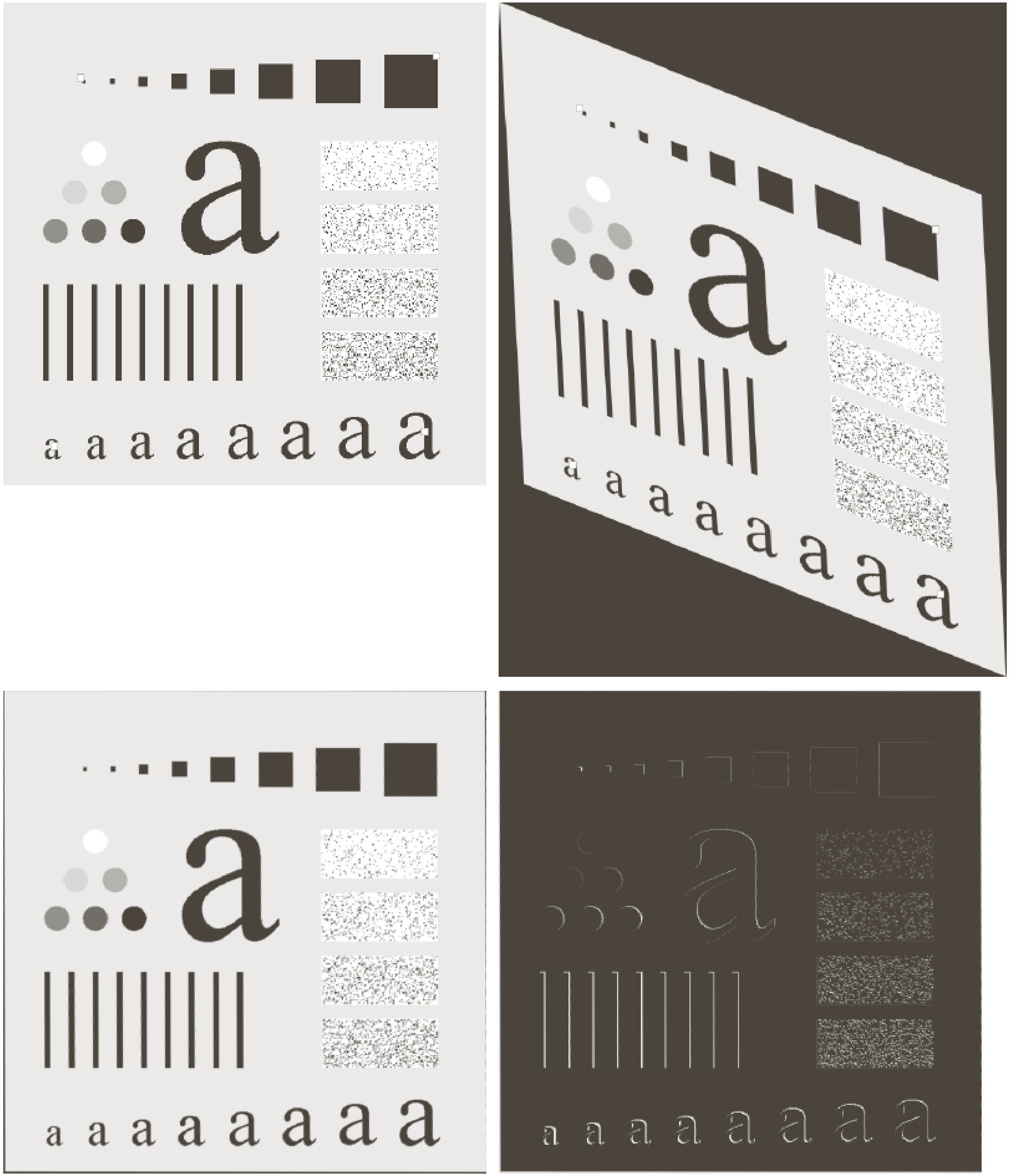


FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

Image Transform

- A particularly important class of 2-D linear transforms, denoted *T(u, v)*

$$T(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)r(x,y,u,v)$$

where $f(x,y)$ is the input image,

$r(x,y,u,v)$ is the *forward transformation kernel*,

variables u and v are the transform variables,

u = 0, 1, 2, ..., M-1 and v = 0, 1, ..., N-1.

Image Transform

- Given T(u, v), the original image f(x, y) can be recoverd using the inverse tranformation of T(u, v).

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u,v)s(x,y,u,v)$$

where $s(x,y,u,v)$ is the *inverse transformation kernel*,

x = 0, 1, 2, ..., M-1 and y = 0, 1, ..., N-1.

Image Transform

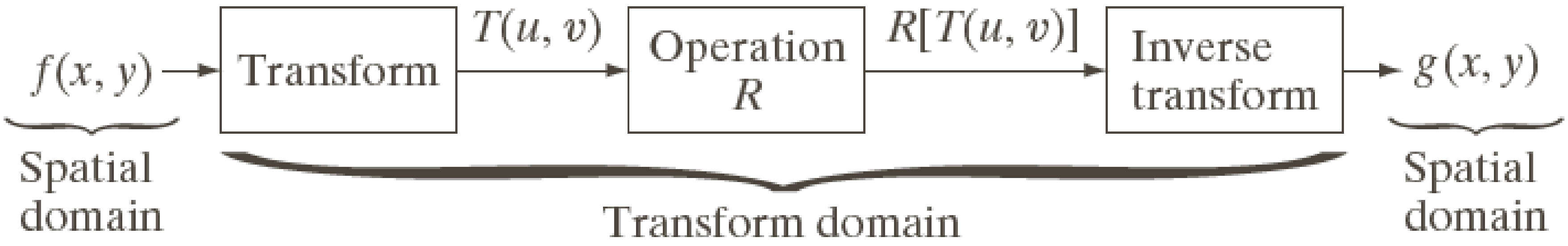


FIGURE 2.39
General approach for operating in the linear transform domain.