Image Restoration and Reconstruction

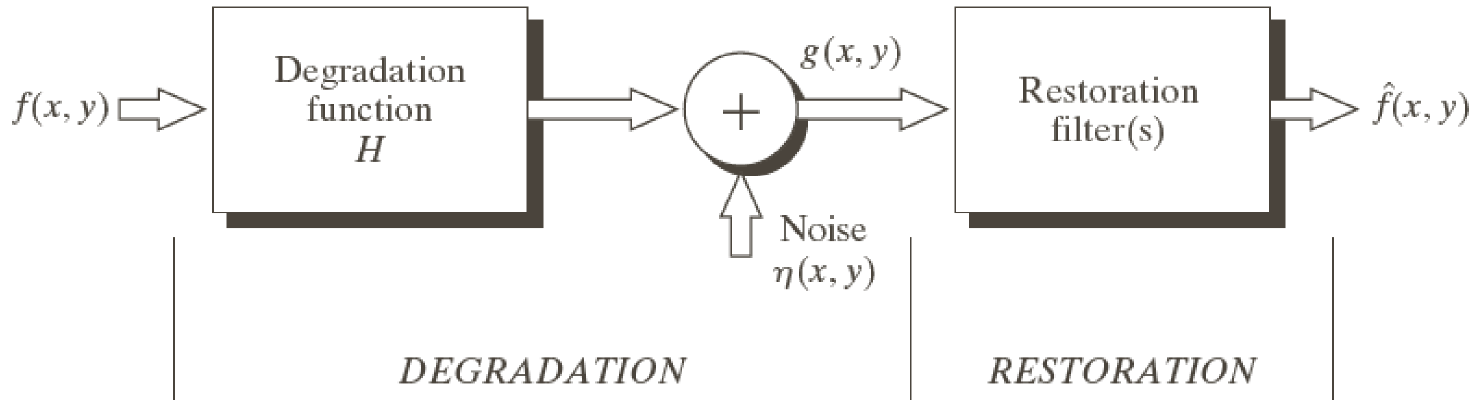
Image Restoration

- Image restoration: recover an image that has been degraded by using a prior knowledge of the degradation phenomenon.

- Model the degradation and applying the inverse process in order to recover the original image.

A Model of Image Degradation/Restoration Process

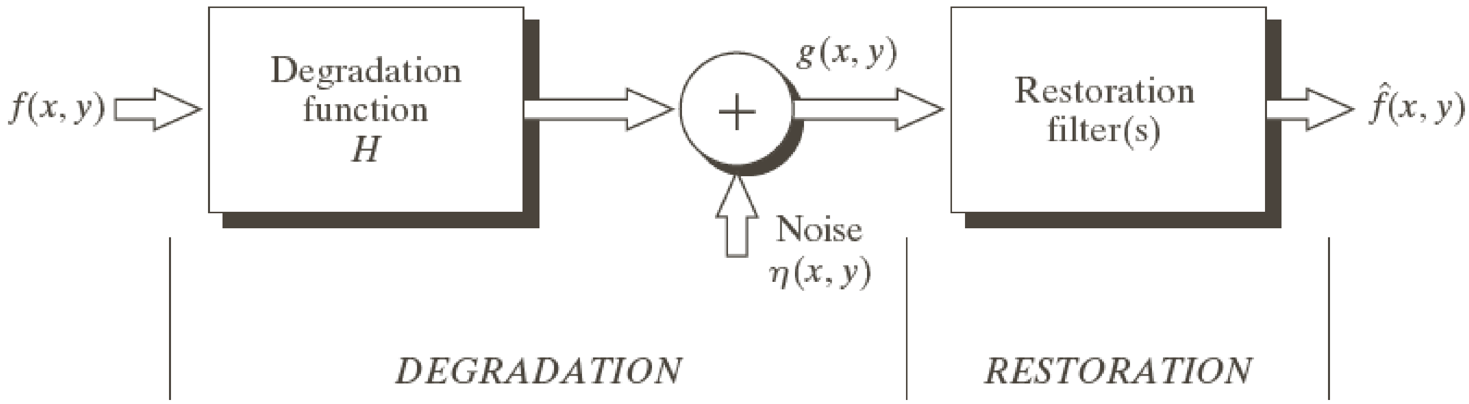
FIGURE 5.1 A model of the image degradation/restoration process.



- Degradation
  - Degradation function H
  - Additive noise $\eta(x, y)$

A Model of Image Degradation/Restoration Process

FIGURE 5.1
A model of the image degradation/restoration process.



If H is a linear, position-invariant process, then the degraded image is given in the spatial domain by

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

Noise Sources

- The principal sources of noise in digital images arise during **image acquisition and/or transmission**

- ✓ Image acquisition

  e.g., light levels, sensor temperature, etc.

- ✓ Transmission

  e.g., lightning or other atmospheric disturbance in wireless network

Noise Models (1)

- With the exception of spatially periodic noise, we assume
  - Noise is independent of spatial coordinates
  - Noise is uncorrelated with respect to the image itself

Noise Models (2)

- **Gaussian noise**
  Electronic circuit noise, sensor noise due to poor illumination and/or high temperature
- **Rayleigh noise**
  Range imaging
- **Erlang (gamma) noise**: Laser imaging
- **Exponential noise**: Laser imaging
- **Uniform noise**: Least descriptive; Basis for numerous random number generators
- **Impulse noise**: quick transients, such as faulty switching

Gaussian Noise

- Gaussian Noise is a statistical noise having a probability density function equal to the **normal distribution**, also known as Gaussian Distribution.
- Random Gaussian function is added to the Image function to generate this noise.
- Source: thermal vibration of atoms and discrete nature of radiation of warm objects.

Gaussian Noise (1)

A noisy image has pixels that are made up of the sum of their original pixel values plus a random Gaussian noise value.

Additive white Gaussian noise is the most common application for Gaussian noise in applications.

The PDF of Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

where, z represents intensity or the grey level,

$\bar{z}$ is the mean (average) value of z

σ is the standard deviation

Gaussian Noise (2)

The PDF of Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

- 70% of its values will be in the range

$$[(\mu-\sigma),(\mu+\sigma)]$$

** $\mu = mean = \bar{z}$
$\sigma = standard\ deviation$

- 95% of its values will be in the range

$$[(\mu-2\sigma),(\mu+2\sigma)]$$

Rayleigh Noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b/4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

Erlang (Gamma) Noise

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = b / a$$

$$\sigma^2 = b / a^2$$

Exponential Noise

The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = 1/a$$

$$\sigma^2 = 1/a^2$$

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Uniform Noise

The PDF of uniform noise is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{for a} \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = (a+b)/2$$

$$\sigma^2 = (b-a)^2/12$$

Impulse (Salt-and-Pepper) Noise

It is also known as salt & pepper noise. This is caused due to sharp & sudden disturbances in the image gray values. Its appearance is randomly scattered white or Black pixels over the image.

There are three types of impulse noises.

Salt Noise: Salt noise is added to an image by the addition of random bright (with 255-pixel value) all over the image.
Pepper Noise: Salt noise is added to an image by the addition of random dark (with 0-pixel value) all over the image.
Salt and Pepper Noise: Salt and Pepper noise is added to an image by addition of both random bright (with 255 pixel value) and random dark (with 0 pixel value) all over the image

Impulse (Salt-and-Pepper) Noise

The PDF of (bipolar) impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

if $b > a$, gray-level b will appear as a light dot, while level a will appear like a dark dot.

If either P_a or P_b is zero, the impulse noise is called *unipolar*

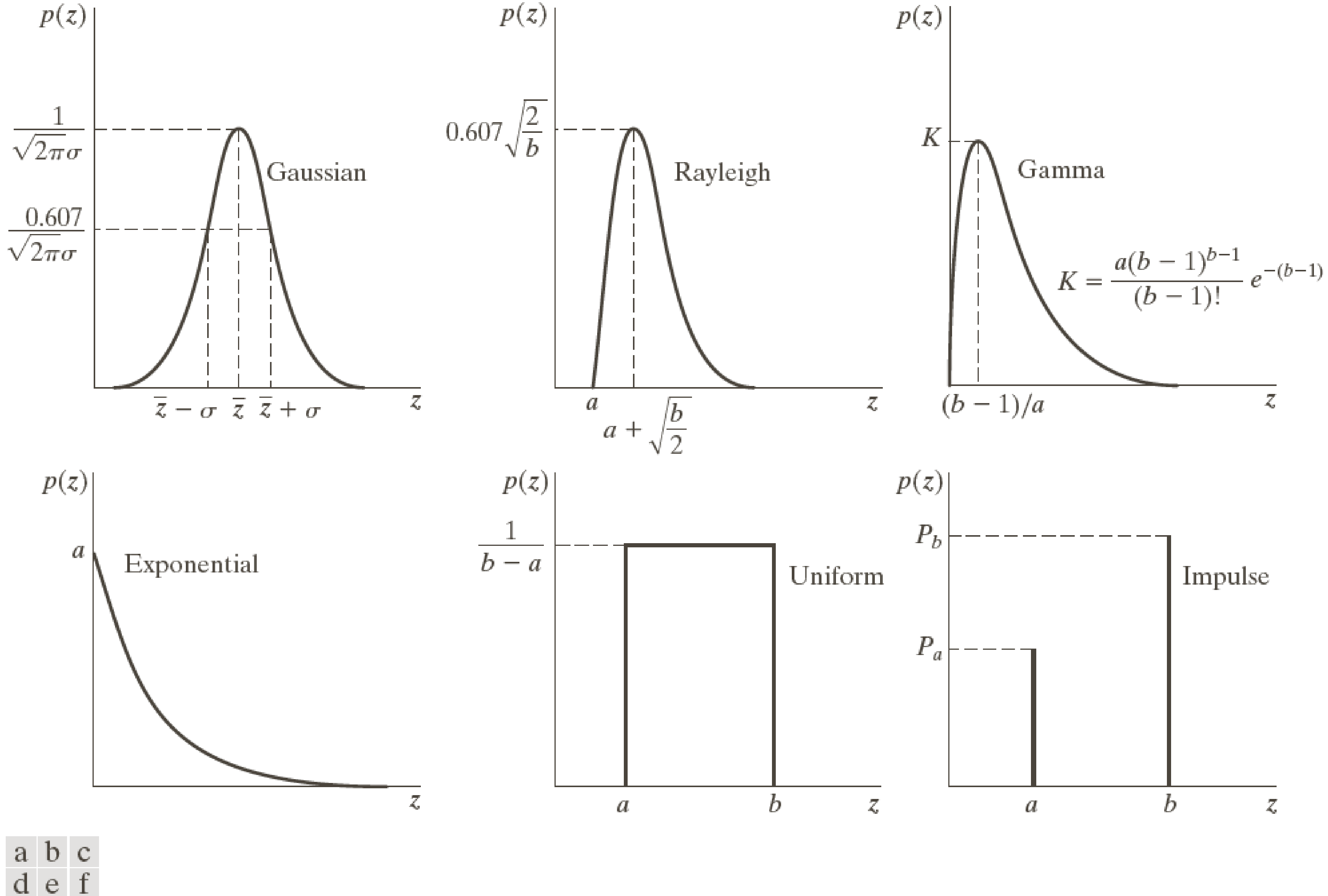
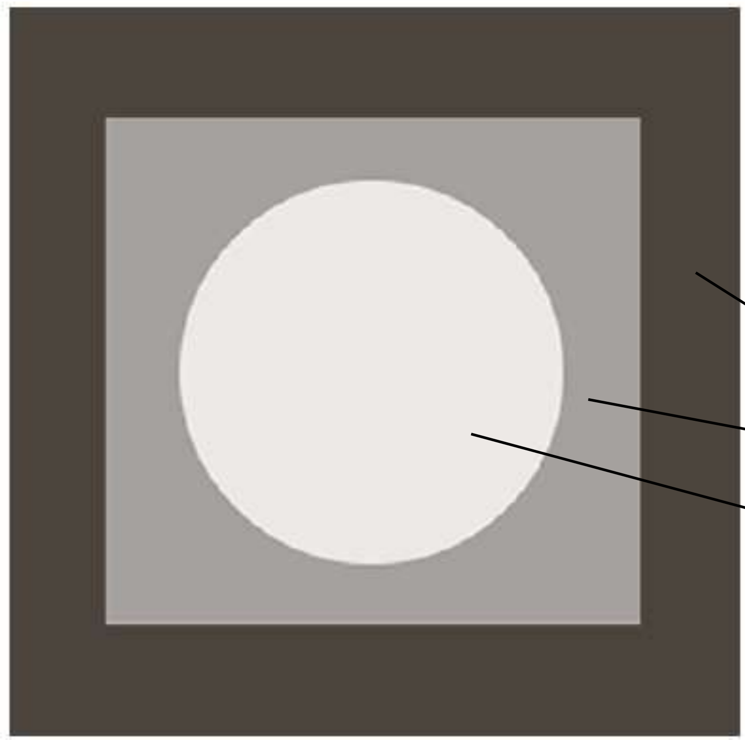


FIGURE 5.2 Some important probability density functions.

Examples of Noise: Original Image



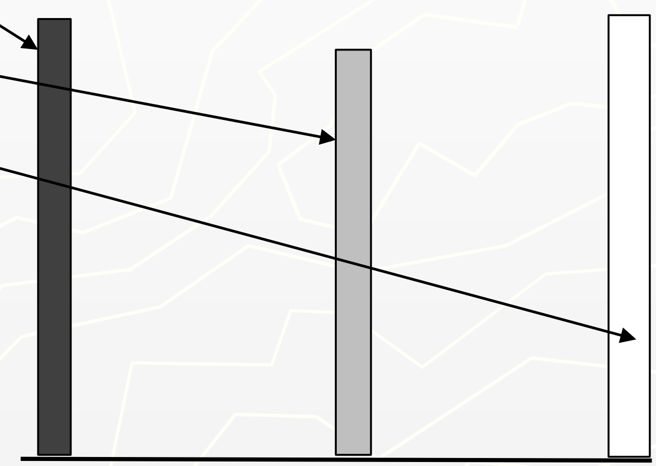


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Examples of Noise: Noisy Images(1)

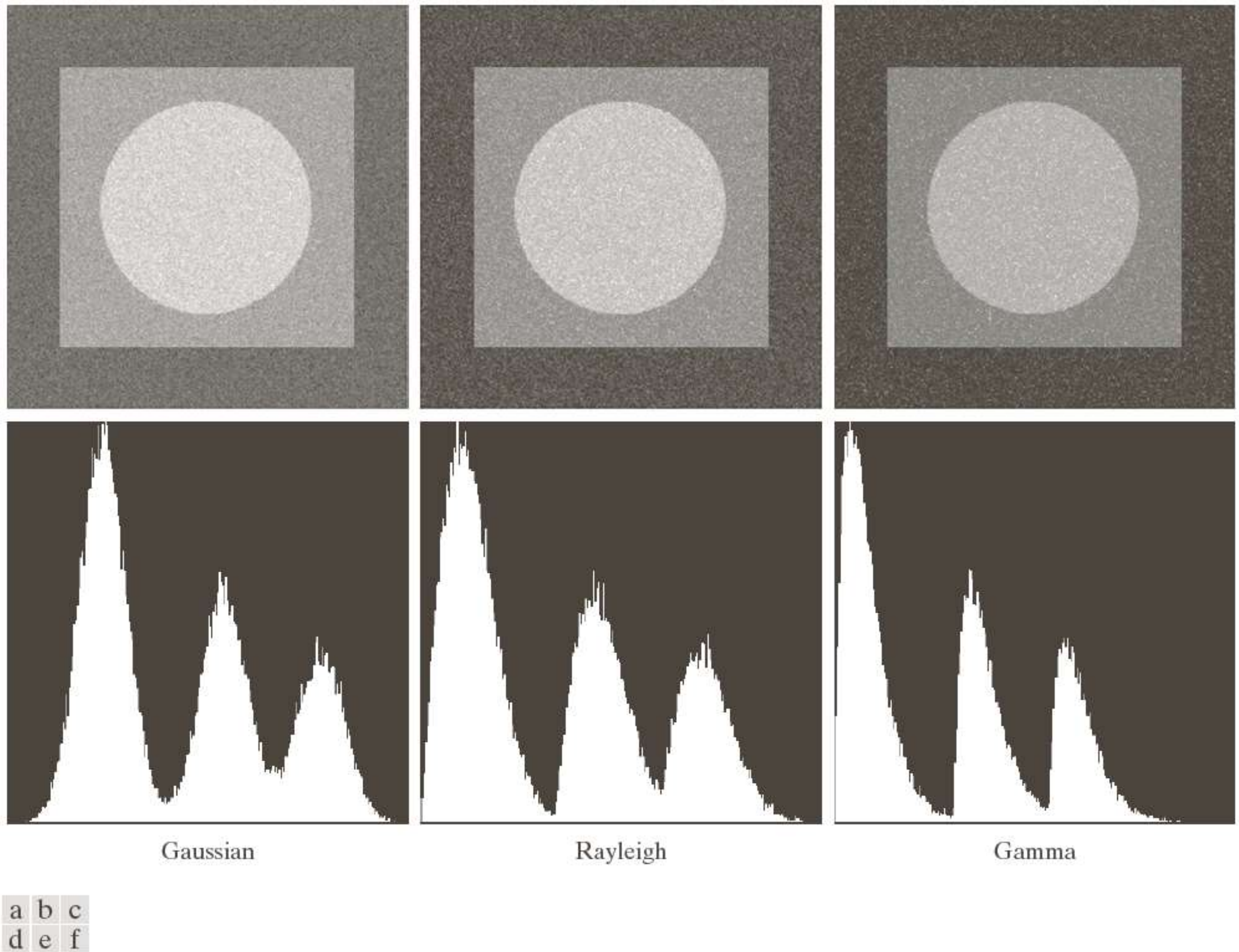


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Examples of Noise: Noisy Images(2)

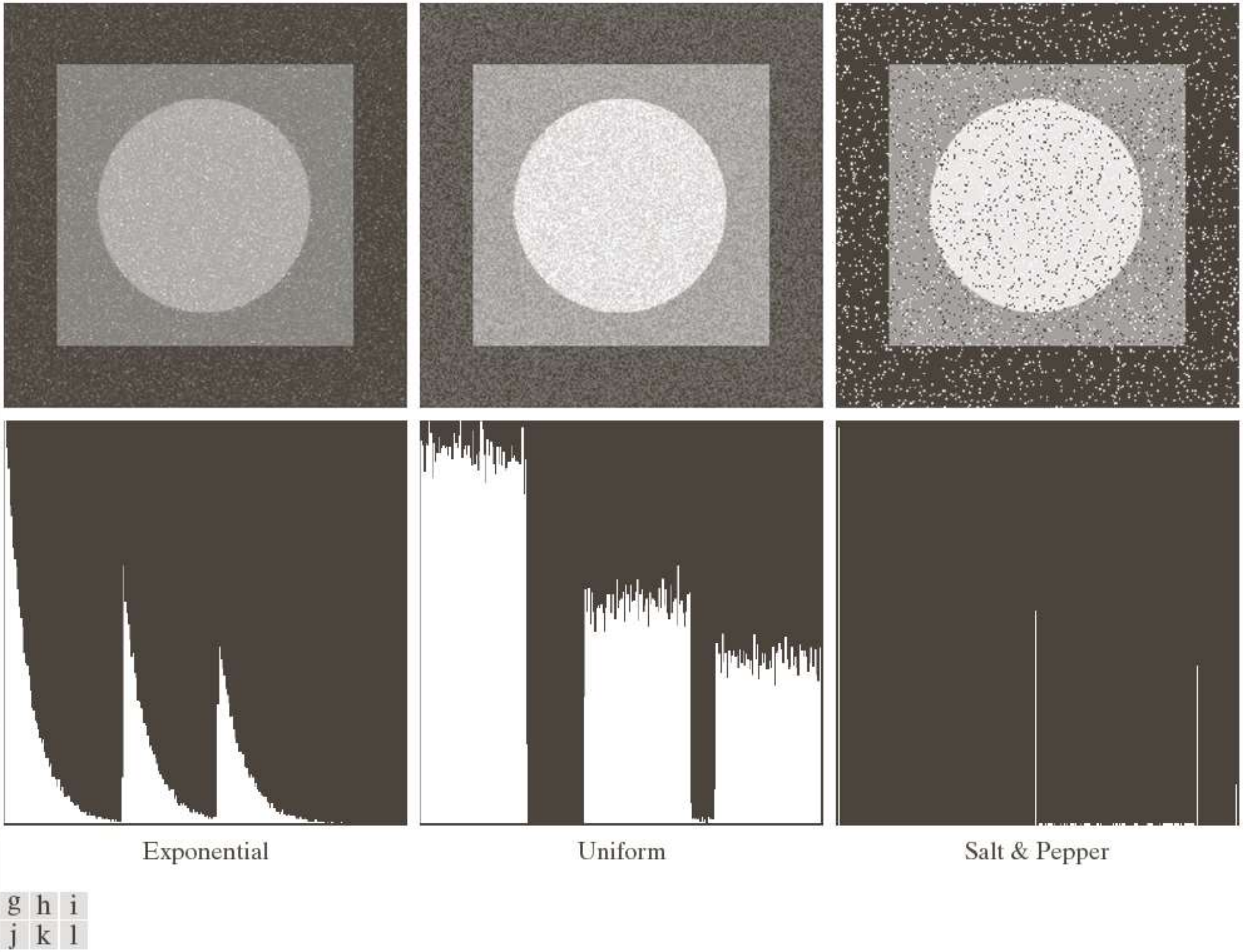


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Estimation of Noise Parameters (1)

The shape of the histogram identifies the closest PDF match

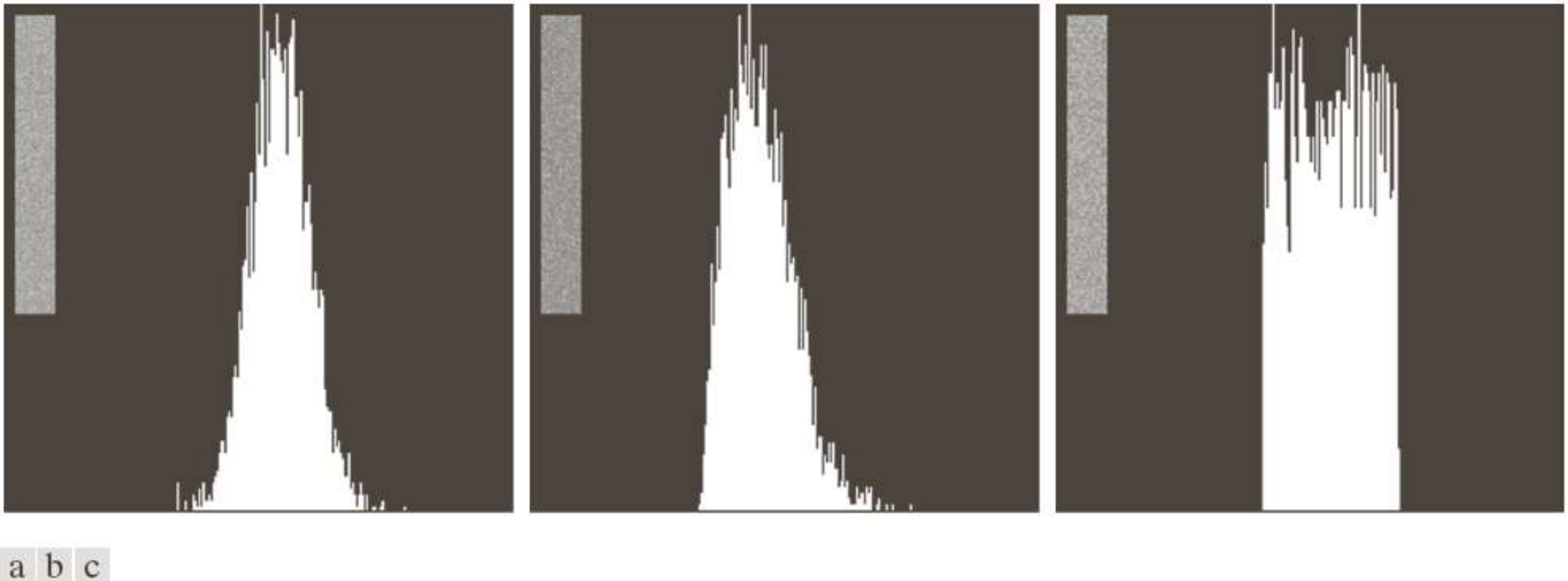


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Spatial Filtering

The spatial Filtering technique is used directly on pixels of an image. Mask is usually considered to be added in size so that it has a specific center pixel. This mask is moved on the image such that the center of the mask traverses all image pixels.

Two types

1. Linear spatial filtering: (use convolution and explicitly use the sum-of-product manner. Mainly used for sharpening (highpass filter) or blurring/smoothing (lawpass filter) the input image)
2. Non-linear spatial filtering: Do not explicitly use the sum-of-product manner. Operation is based conditionally on the neighboring pixels. Can be used for noise reduction.

Spatial Filtering: Mean Filters (1)

- **Smoothing Spatial Filter:** Smoothing filter is used for blurring and noise reduction in the image. Blurring is pre-processing steps for removal of small details and Noise Reduction is accomplished by blurring.

Let S_{xy} represent the set of coordinates in a rectangle subimage window of size $m \times n$, centered at (x, y).

Arithmetic mean filter

$$f(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Spatial Filtering: Mean Filters (2)

Geometric mean filter

$$f(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Generally, a geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process

The geometric mean filter is better at **removing Gaussian-type noise and preserving edge features than the arithmetic mean filter**.

Spatial Filtering: Mean Filters (3)

Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

It works well for salt noise, but fails for pepper noise.
It does well also with other types of noise like Gaussian noise.

Spatial Filtering: Mean Filters (4)

Contraharmonic mean filter

$$f(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

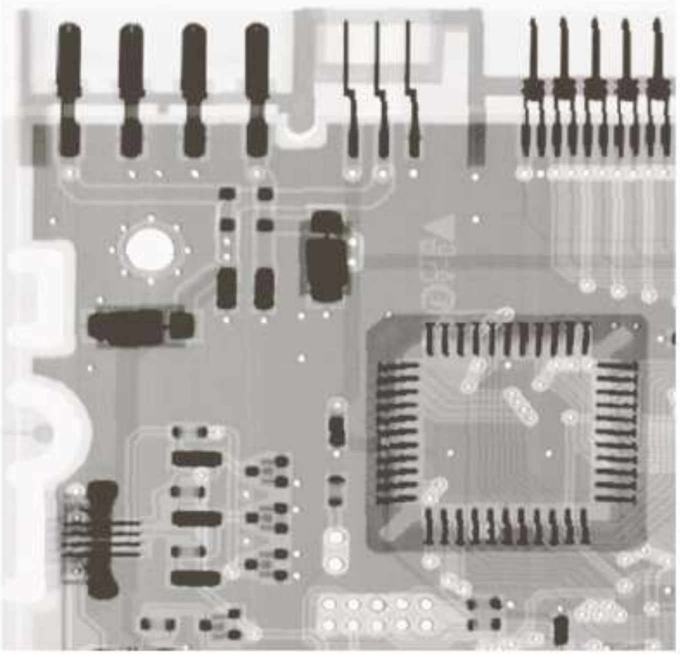
Q is the order of the filter.

It is well suited for reducing the effects of salt-and-pepper noise. Q>0 for pepper noise and Q<0 for salt noise.

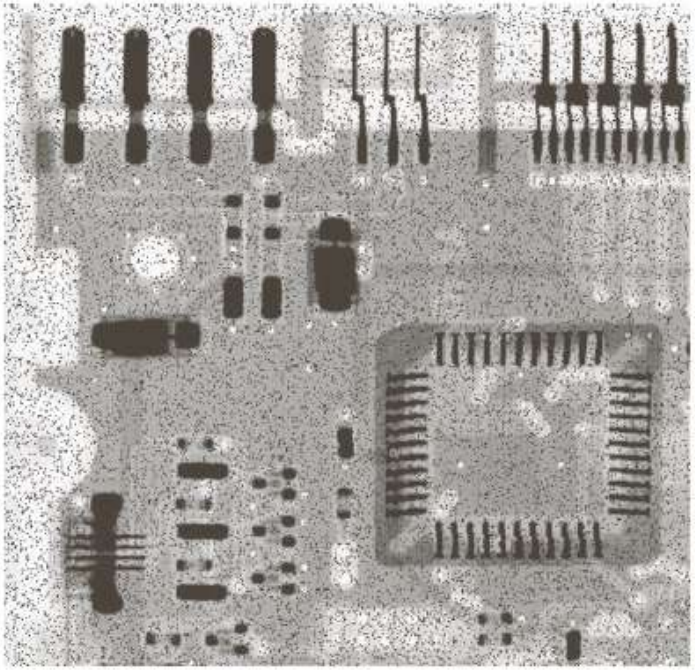
Spatial Filtering: Example (1)

a	b
c	d

FIGURE 5.7
(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



Spatial Filtering: Example (2)



a	b
c	d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Spatial Filtering: Order-Statistic (non-linear) Filters (1)

Median filter

$$f(x,y) = \underset{(s,t)\in S_{xy}}{median}\left\{g(s,t)\right\}$$

Max filter

$$f(x,y) = \max_{(s,t)\in S_{xy}}\left\{g(s,t)\right\}$$

Min filter

$$f(x,y) = \min_{(s,t)\in S_{xy}}\left\{g(s,t)\right\}$$

Spatial Filtering: Order-Statistic Filters (2)

Midpoint filter

$$f(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\} \right]$$

Spatial Filtering: Order-Statistic Filters (3)

Alpha-trimmed mean filter

$$f(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} \{g_r(s,t)\}$$

It is a hybrid of the mean and median filters. The basic idea behind the filter is for any element of the signal (image) to look at its neighborhood, discard the most atypical elements and calculate the mean value using the rest of them.

We delete the $d/2$ lowest and the $d/2$ highest intensity values of $g(s,t)$ in the neighborhood S_{xy}. Let $g_r(s,t)$ represent the remaining $mn-d$ pixels.

a	b
c	d

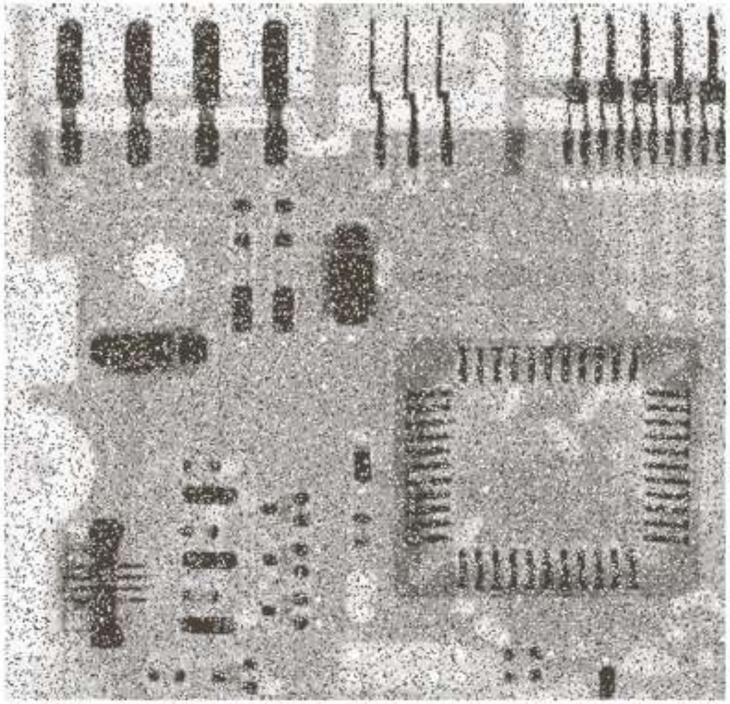
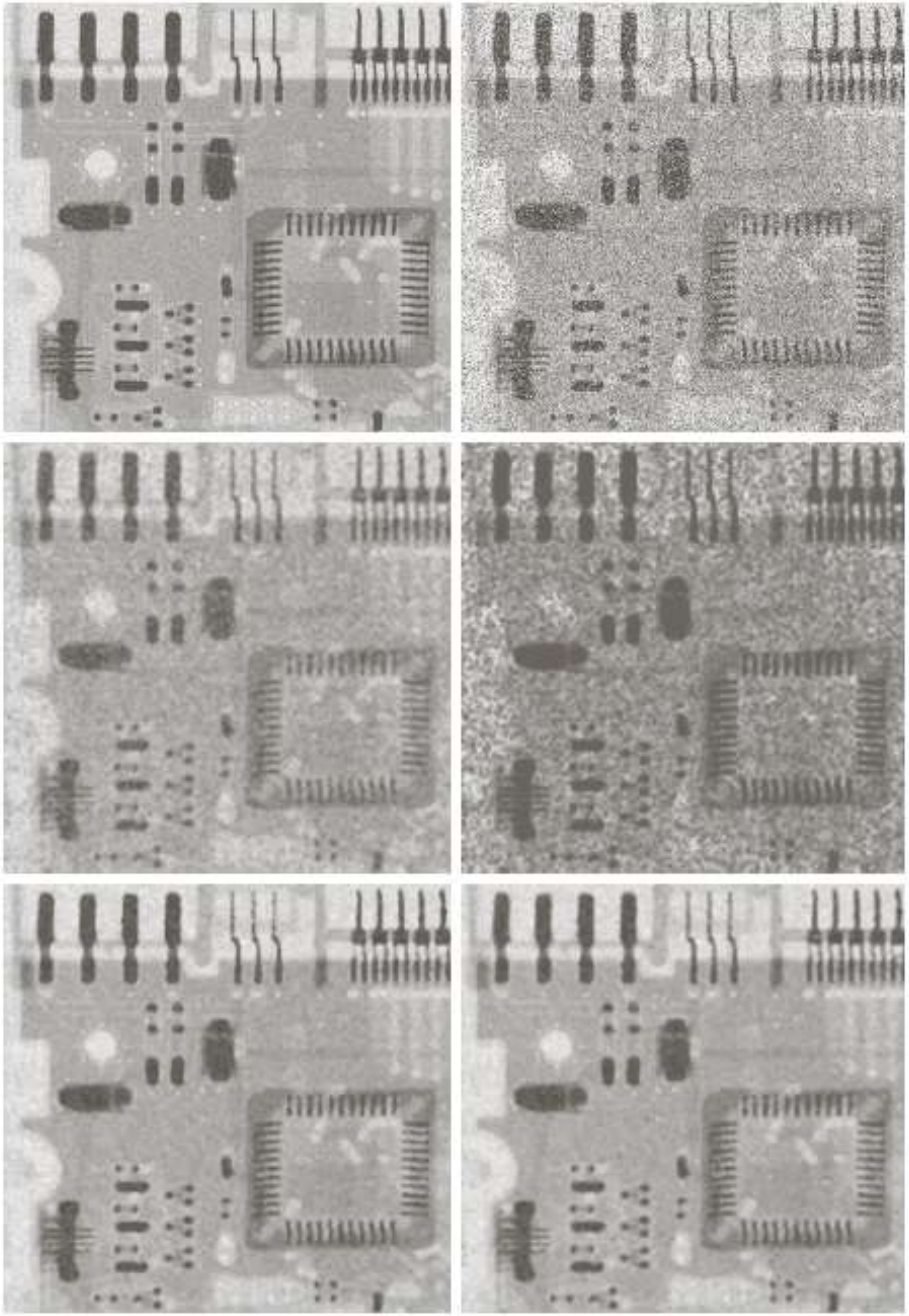


FIGURE 5.10
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size 3×3.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.

a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.



a	b
c	d
e	f

FIGURE 5.12
(a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Spatial Filtering: Adaptive Filters (1)

Adaptive filters

The behavior changes based on statistical characteristics of the image inside the filter region defined by the mxn rectangular window.

The performance is superior to that of the filters discussed

Adaptive Filters:

Adaptive, Local Noise Reduction Filters (1)

S_{xy}: local region

The response of the filter at the center point (x,y) of S_{xy} is based on four quantities:

(a) $g(x, y)$, the value of the noisy image at (x, y);

(b) σ_{η}^{2}, the variance of the noise corrupting $f(x, y)$ to form $g(x, y)$;

(c) m_L, the local mean of the pixels in S_{xy};

(d) σ_{L}^{2}, the local variance of the pixels in S_{xy}.

Adaptive Filters:

Adaptive, Local Noise Reduction Filters (2)

The behavior of the filter:

(a) if σ_η^2 is zero, the filter should return simply the value of $g(x, y)$.

(b) if the local variance is high relative to σ_η^2, the filter should return a value close to $g(x, y)$;

(c) if the two variances are equal, the filter returns the arithmetic mean value of the pixels in S_{xy}.

Adaptive Filters:
Adaptive, Local Noise Reduction Filters (3)

An adaptive expression for obtaining $f(x, y)$ based on the assumptions:

$$f(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right]$$

a	b
c	d

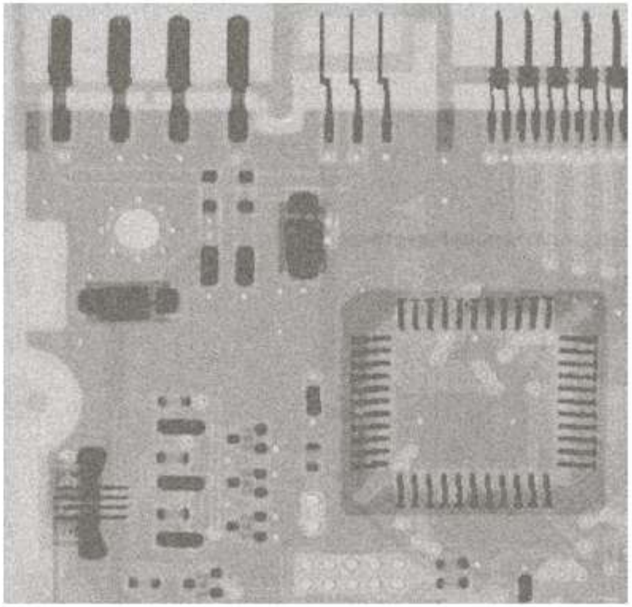


FIGURE 5.13
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.
(b) Result of arithmetic mean filtering.
(c) Result of geometric mean filtering.
(d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Adaptive Filters:
Adaptive Median Filters (1)

The notation:

$z_{\min}$ = minimum intensity value in S_{xy}

$z_{\max}$ = maximum intensity value in S_{xy}

z_{med} = median intensity value in S_{xy}

z_{xy} = intensity value at coordinates (x, y)

$S_{\max}$ = maximum allowed size of S_{xy}

Adaptive Filters:

Adaptive Median Filters (2)

The adaptive median-filtering works in two stages:

Stage A:

$A1 = z_{med} - z_{min}$; $A2 = z_{med} - z_{max}$

if A1>0 and A2<0, go to stage B

Else increase the window size

if window size $\leq S_{max}$, repeat stage A; Else output z_{med}

Stage B:

$B1 = z_{xy} - z_{min}$; $B2 = z_{xy} - z_{max}$

if B1>0 and B2<0, output z_{xy}; Else output z_{med}

Adaptive Filters:
Adaptive Median Filters (2)

The adaptive median-filtering works in two stages:

Stage A:

$A1 = z_{med} - z_{min}$; $A2 = z_{med} - z$

if A1>0 and A2<0, go to stage B

Else increase the window size

if window size $\leq S_{max}$, repeat stage A; Else output z_{med}

The median filter output is an impulse or not

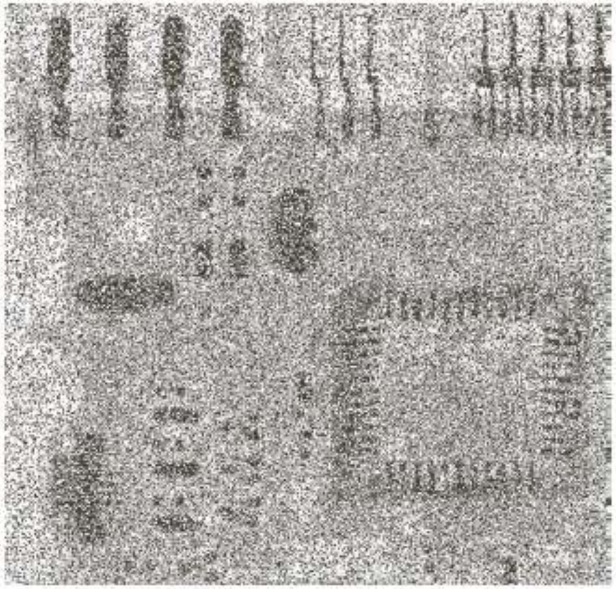
Stage B:

$B1 = z_{xy} - z_{min}$; $B2 = z_{xy} - z$

if B1>0 and B2<0, output z_{xy}; Else output z_{med}

The processed point is an impulse or not

Example:
Adaptive Median Filters



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Inverse Filtering

An estimate of the transform of the original image

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$\hat{F}(u,v) = \frac{F(u,v)H(u,v) + N(u,v)}{H(u,v)}$$

$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Inverse Filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

1. We can't exactly recover the undegraded image because $N(u,v)$ is not known.

Some Measures (1)

Singal-to-Noise Ratio (SNR)

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} |F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} |N(u,v)|^2}$$

This ratio gives a measure of the level of information bearing singal power to the level of noise power.

Some Measures (2)

Mean Square Error (MSE)

$$\text{MSE}=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y)-f(x,y)\right]^2$$

Root-Mean-Sqaure-Error (RMSE)

$$\text{RMSE}=\frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} f(x,y)^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} | f(x,y)-f(x,y)|^2}$$